DHT Based 4 QAM OFDM Baseband System and Channel Estimation

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Abstract **- This paper presents the simulation of 4 Quadrature Amplitude Modulation (QAM) orthogonal frequency division multiplexing (OFDM) baseband system and channel estimation which uses inverse discrete Hartley transform (IDHT) and discrete Hartley transform (DHT). As the calculation of DHT and IDHT involves real operations hence the computational complexities are less as compared to DFT and IDFT. Moreover as IDHT is same as DHT hence we can use same hardware for both, while the DFT and IDFT require separate hardware to implement. As compared to DFT based OFDM system, the simulated DHT based OFDM system achieves approximately the same transmission performance with less computational complexity and hardware requirements.**

Index Terms **- Channel estimation, DHT, DFT, QAM, OFDM**

I Introduction

Frequency division multiplexing (FDM) is used to transmit multiple signals simultaneously over a wired or a wireless system. Each signal is limited by a specific frequency band, and is modulated by a data stream. Orthogonal Frequency Division Multiplexing (OFDM) [7, 8] is a special case of this, where the data is distributed over a large number of carriers that are 'orthogonal' to each other. OFDM is spectrally efficient compared to the conventional FDM system, since it does not need any guard bands between adjacent channels. This orthogonality property is the heart of OFDM, since the interference due to other carriers is prevented, when the receiver demodulates a particular carrier.

The OFDM system avoids the tracking of a time varying channel by the use of differential phase shift keying (DPSK) in. However this will limit the number of bits per symbol and results in 3 db loss in signal-to-noise ratio (SNR) [2]. If the receiver contains a channel estimator, multipath signaling scheme can be used. OFDM system offers high bit rate transmission over a frequency fading channel due to use of spectrally efficient quadrature amplitude modulation (QAM). Because of high efficiency with no inter-symbol-interference (ISI), OFDM has been standardized or extensively investigated for various applications like digital radio applications and wireless communications [1].

Implementation of the DFT based OFDM system requires the complex calculation of a long length IDFT and DFT on the transmitter and receiver side of the OFDM system. Such a long length IDFT and DFT calculation requires a huge number of complex multiplications and additions.

In this paper we simulated a discrete Hartley transform (DHT) [10] based OFDM system which uses IDHT and DHT on the transmitter and receiver side. DHT is purely real transform. The calculation of DHT involves only real multiplications and additions and it is having identical inverse. Here, we present a comparative analysis of DFT and DHT based OFDM system. The objective is to examine the computational complexities and bit error rate (BER) performance after channel estimation of DHT and DFT based OFDM system [12]. The rest of the work is organized as follows. In Section II DHT based OFDM system model is described. Section III includes the channel estimation algorithms. Section IV contains performance evaluation results obtained by means of simulations. This is followed by conclusions in Section V.

The similar type of work might have been done by someone else too but the methodology adapted in this paper is totally new up to best of our knowledge and data available to us.

II DHT Based OFDM System Model

The OFDM system is modeled employing the following assumptions.

1) Accurate time & frequency synchronization at the receiver.
2) The channel impulse response length (L_t) is smaller than the cyclic prefix (CP) length of the OFDM symbol in order to avoid inter-block interference & preserve orthogonality of the OFDM symbol.

We will consider the system shown in Fig. 1, where x_k are the transmitted symbols, $g(t)$ is the channel impulse response, $\hat{n}(t)$ is the white complex Gaussian channel noise and y_k are the received symbols. The transmitted symbols x_k are taken from a multi-amplitude signal constellation. The D/A and A/D converters contain ideal low pass filter with bandwidth $1/T_s$, where T_s is the sampling interval. We treat the channel impulse response $g(t)$ as a time limited pulse response of the form [3]

$$g(t) = \sum_{m=0}^{L_t - 1} \alpha_m \partial(t - \tau_m T_s) \qquad (1)$$

where the amplitudes α_m are complex valued and $0 \le \tau_m \le T_G$, i.e., the entire impulse response lies inside

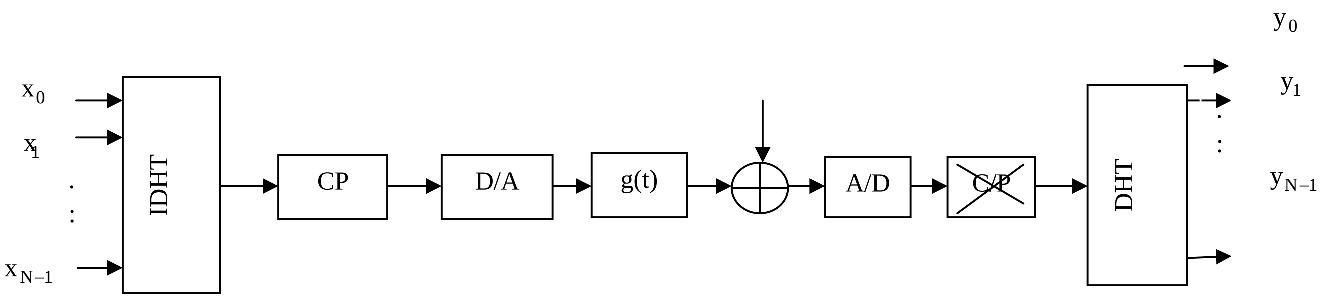

Fig. 1 Baseband OFDM System Model

the guard space. Let X_k be an N-point conjugate symmetric complex valued sequence such that $0 \le k \le N-1$ and $X_{N-k} = X_k^*$. The N –point IDFT of X_k is given as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k . W_N^{nk}, n = 0,1,\ldots,N-1 \qquad (2)$$

where $W_N = e^{j2\pi/N}$ and x_n is a real valued sequence. If $x = [x_{N-1}, x_{N-2}, \ldots, x_1, x_0]^T$ and $X = [X_{N-1}, X_{N-2}, \ldots, X_1, X_0]^T$ then we can express equation (2) as

$$x = QX \qquad (3)$$

where Q is an IDFT matrix and can be expressed as

$$Q = \frac{1}{\sqrt{N}} \begin{bmatrix} W_N^{(N-1)^2} & W_N^{(N-1)(N-2)} & \cdots & W_N^{N-1} & 1 \\ W_N^{(N-1)(N-2)} & W_N^{(N-2)^2} & \cdots & W_N^{N-2} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ W_N^{N-1} & W_N^{N-2} & \cdots & W_N^{1} & 1 \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \qquad (4)$$

Similarly, The DHT of N-point real sequence $s = [s_{N-1}, s_{N-2}, \ldots, s_1, s_0]^T$ is given by

$$S_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_n H_N^{kn}, k = 0,1,2,\cdots,N-1 \qquad (5)$$

Where $S = [S_{N-1}, S_{N-2}, \cdots, S_1, S_0]^T$ is the transform output vector and $H_N^k = \sin(2\pi k/N) + \cos(2\pi k/N)$. We can express the equation (5) as

$$S = Hs \qquad (6)$$

where H can be expressed as

$$H = \frac{1}{\sqrt{N}} \begin{bmatrix} H_N^{(N-1)^2} & H_N^{(N-1)(N-2)} & \cdots & H_N^{N-1} & 1 \\ H_N^{(N-1)(N-2)} & H_N^{(N-2)^2} & \cdots & H_N^{N-2} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ H_N^{N-1} & H_N^{N-2} & \cdots & H_N^{1} & 1 \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \qquad (7)$$

As we know that DHT has identical inverse so IDHT and DHT matrices are same i.e. $H = H^{-1}$ so

$$s = H^{-1}S = HS \qquad (8)$$

The channel output of IDHT modulated data in DHT based OFDM is given as [9]

$$r = H\Phi S + n \qquad (9)$$

where $r = [r_{N-1}, r_{N-2}, \cdots, r_1, r_0]^T$ and Φ is given as

$$\Phi = \begin{bmatrix} a_1 & 0 & \cdots & 0 & \cdots & 0 & -b_1 & 0 \\ 0 & \ddots & \ddots & \vdots & \ddots & \iddots & 0 & 0 \\ \vdots & \ddots & a_{N/2-1} & 0 & -b_{N/2-1} & \iddots & \vdots & \vdots \\ 0 & \cdots & 0 & b_0 & 0 & \cdots & 0 & 0 \\ \vdots & \iddots & b_{N/2-1} & 0 & a_{N/2-1} & \ddots & \vdots & \vdots \\ 0 & \iddots & \iddots & \vdots & \ddots & \ddots & 0 & 0 \\ b_1 & 0 & \cdots & 0 & \cdots & 0 & a_1 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & a_0 \end{bmatrix} \quad (10)$$

with a_i and b_i are being real numbers. The output of DHT based OFDM system after N-point DHT is given by [9]

$$y = Hr = \Phi S + Hn \quad (11)$$

The system modeled using N-point DHT can also be expressed as

$$y = DHT_N\left(IDHT_N(x) \otimes \frac{g}{\sqrt{N}} + \hat{n} \right) \quad (12)$$

Where $\otimes$ denotes the cyclic convolution, $x = [x_0, x_1 ... x_{N-1}]^T$, $y = [y_0, y_1 ... y_{N-1}]^T$, $\hat{n} = [\hat{n}_1, \hat{n}_2 ... \hat{n}_{N-1}]^T$ is a vector of independent identically distributed complex Gaussian variables, and $g = [g_0, g_1 ... g_{N-1}]^T$ is determined by cyclic equivalent of sinc-functions.

The system described by (12) can be written as a set of N independent Gaussian channels,

$$y_K = h_K x_K + n_K \quad (13)$$

Where K= 0 . . . , N-1

Where h_K is complex channel attenuation given by $h = [h_0, h_1 ... h_{N-1}]^T = DHT_N(g)$ and $x = [x_0, x_1 ... x_{N-1}]^T$ $n = [n_0, n_1 ... n_{N-1}]^T = DHT_N(n)$ is an independent identically distributed complex zero mean Gaussian noise vector.As a matter of convenience, we write (13) in matrix notation

$$y = XFg + n \quad (14)$$

Where **X** is a matrix with elements of **x** on its diagonal and **F** is N×N DHT matrix with

$$H_N^k = \sin(2\pi k / N) + \cos(2\pi k / N) \quad (15)$$

III Channel Estimation

Efficient channel estimation strategies are required for coherent detection & decoding. Adaptive estimation of the channel is necessary before the demodulation of OFDM signals since the channel is frequency and time varying.

A. *Least Squares Estimators*

The LS estimator [6] for the cyclic impulse response g minimizes $(y - XFg)^H (y - XFg)$ and generates

$$\hat{h}_{LS} = FQ_{LS}F^H X^H y \quad (16)$$

where

$$Q_{LS} = (F^H X^H XF)^{-1} \quad (17)$$

and hence

$$\hat{h}_{LS} = X^{-1} y \quad (18)$$

where $(.)^H$ denotes hermitian transposition.

B. *Modified Least Squares Estimators*

In the above case the size of Q is same as that of X**,** but it has been found out [3] that first $L = \frac{T_c}{T_s}$ taps or nearby taps of g contains the most of the energy. By considering only the taps with the significant energy in modified LS helps in improving the mean-square error for a range of SNRs. Using channel statistics and taking only the first L taps of **g** into account, the modified LS estimator becomes [4]

$$\hat{h}_{LS} = TQ'_{LS}T^H X^H y \quad (19)$$

where

$$Q'_{LS} = (T^H X^H XT)^{-1} \quad (20)$$

IV Results And Performance Analysis

We have carried out the simulation using the pedestrian-B (PED-B) channel with the specifications : OFDM system with DHT size: N_{DHT}=512, cyclic prefix length: N_{CP}=64, clock rate: F_s=24 Mega samples/sec, 1 sample=8 bits, subcarrier spacing: F_s/N_{DHT}=3.90625 kHZ, useful symbol time T_b= 256 µs, cyclic prefix time: $T_G=T_b/8$=32µs. OFDM symbol time: $T_s=T_b+T_G$=288µs, sampling time: $T_b/N_{DHT}=1/F_s$=0.5µs, sampling frequency: F_s=2MHz, Bandwidth: BW=3.90625*512=2MHz. OFDM frame time = 3.168 ms.

A. Pedestrian-B channel

Tap Positions (µs)	Tap Gain (dB)
1	0
2	-1
5	-9
7	-10
12	-15
19	-20

It is quite clear from the defination of DHT and DFT that computational complexities involved in implementing DHT are quite lesser than that in DFT implementation although both gives same transmission efficiency [9, 11]. As the DHT has identical inverse so same hardware can be used for DHT and IDHT hence the implementation DHT based OFDM system is cost effective as compared to DFT based OFDM system. We have analysed the peformance result of DHT based OFDM system using channel estimation graph and BER Versus SNR. Also compared the results with that of DFT based OFDM system.

B. Channel Estimation Plot

Channel estimation for both the DHT and DFT based OFDM system has been done using least squares (LS) channel estimator and modified least square (MLS) channel estimator and compared with the actual channel. Fig. 2 shows the channel estimation plot for DFT based OFDM system and Fig. 3 shows the channel estimation plot for DHT based OFDM system. It is clear from the Fig. 2 and Fig. 3 shows that LS and MLS channel estimators estimates the channel equally well and better at some points in case of DHT based OFDM system in comparison to DFT based OFDM system

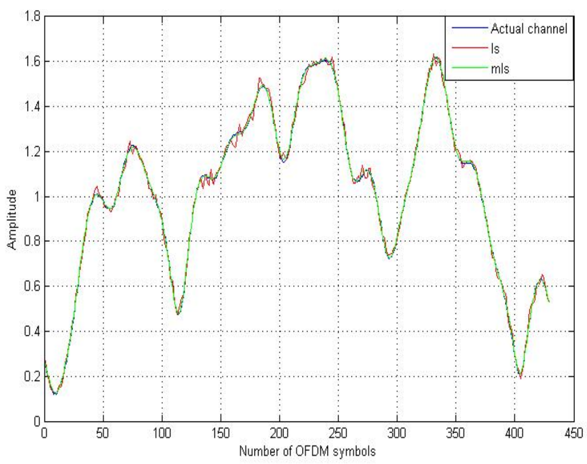

Fig. 2 Channel estimation plot for DFT based OFDM system

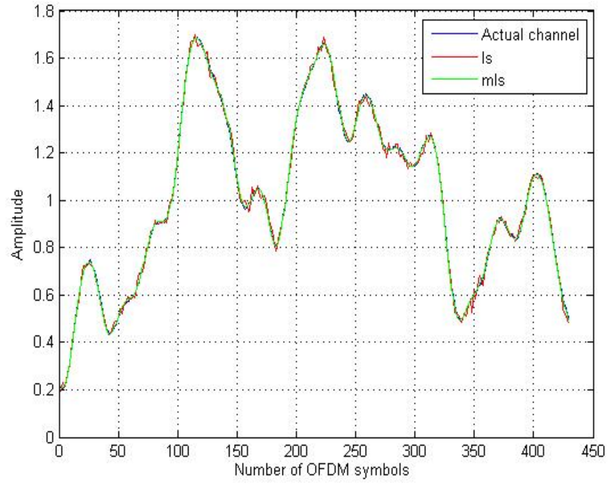

Fig. 3 Channel estimation plot for DHT based OFDM system

C. BER Versus SNR Curve

The performance of the proposed DHT based OFDM system has been analysed by plotting bit error rate (BER) versus SNR curve after channel estimation and compared it with that of DFT based OFDM system. Fig. 4 shows BER versus SNR for DHT based system after channel estimation. . Fig. 5 shows BER versus SNR for DFT based system after channel estimation. Table 1 shows BER comparison of LS and MLS channel estimator for DHT and DFT based OFDM system at different SNR. It can be observed clearly from the Fig. 4 and Fig. 5 that at different SNR, the bit error rate performance for DHT and DFT based OFDM is almost same or better in case of DHT based OFDM system which justifies the importance of proposed method.

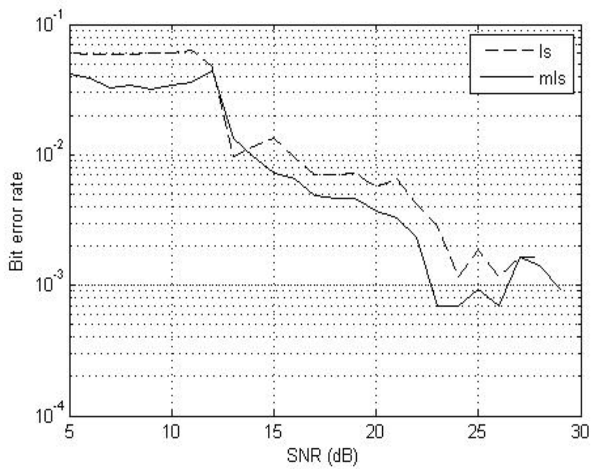

Fig.4 BER Versus SNR curve for DHT based system after channel estimation

SNR (dB)	BER in DFT Based OFDM System for Least Squares Channel estimator	BER in DHT Based OFDM System for Least Squares Channel estimator	BER in DFT Based OFDM System for Modified Least Squares Channel estimator	BER in DHT Based OFDM System for Modified Least Squares Channel Estimator
5.0	0.682	0.06028	0.682	0.04029
10.0	0.4761	0.6021	0.4642	0.03458
15.0	0.2068	0.01355	0.1949	0.007423
20.0	0.02196	0.005607	0.01572	0.003738
25.0	0.008645	0.001869	0.005374	0.0009346

Table 1. Showing BER comparison of LS and MLS channel estimator for DHT and DFT based OFDM system at different SNRs

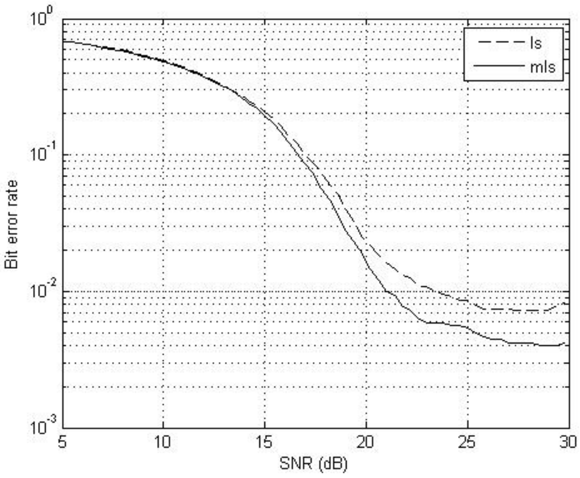

Fig.5 BER Versus SNR curve for DFT based system after channel estimation

V CONCLUSIONS

We have simulated DHT based OFDM system. The DHT is real valued transform and having identical inverse while the DFT is a complex transform and not having identical inverse. So the DHT based OFDM is having less computational complexities than DFT based OFDM system. As DHT is having identical inverse so implementation of DHT based results in reduction in hardware requirement as the same hardware can be used for inverse DHT on receiver side. The simulation results shows that DHT based OFDM system is having same transmission efficiency, BER peformance as that of DFT based OFDM system with less computational complexities & hardware requirement.

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