

# Non-linear Adaptive Control System for an Underactuated Autonomous Underwater Vehicle using Dynamic State Feedback

Santhakumar, M. and Asokan, T.

Department of Engineering Design, Indian Institute of Technology Madras, Chennai, India

Email: santha\_radha@yahoo.co.in, asok@iitm.ac.in

**Abstract**— An adaptive, non-linear control strategy for underactuated Autonomous Underwater Vehicle (AUV) is proposed in this paper. In order to improve the autonomy of AUVs, it is important to develop intelligent control systems which can reduce the effects of coupled and non-linear characteristics of the vehicles. As most of the AUVs are under actuated systems, classical control scheme fails due to the non-integrable acceleration constraints on the vehicle. A dynamic state feedback law is proposed here to overcome this and to reduce the modelling errors arising out of various assumptions on the vehicle dynamics. In addition, proposed method can work satisfactorily under commonly encountered uncertainties such as ocean current and parameter variations. The system states are stabilized by forcing the tracking errors to an arbitrarily small neighborhood of zero by using backstepping technique and the tracking error dynamics. The proposed strategy has been tested through simulations, for a flat-fish type AUV. A comparison of the results for conventional method and the proposed method is also presented.

**Index Terms**—underwater vehicle, adaptive control, dynamic state feedback, underactuated control;

## I. INTRODUCTION

Modern developments in the fields of control, sensing, and communication have made increasingly complex and dedicated underwater robot systems a reality. Used in a highly hazardous and unknown environment, the autonomy and dynamics of the robots is the key to mission success. Navigation and control of underwater robots/vehicles for successful execution of subsea tasks has been a hot area of research in the last few decades, especially for developing advanced control strategies for Autonomous Vehicles. Recent developments in this area are well summarized in [1, 2]. Design of controllers for AUVs is an extremely difficult task due to the uncertainty of hydrodynamic parameters. Additionally, most of the AUVs are under actuated systems [1, 2, 3], and this imposes non-integrable acceleration constraints on the vehicle where the classical control scheme fails. The problem of designing a stabilizing feedback controller for underactuated systems is a challenging one since the system is not stabilizable by a smooth static state feedback law [4]. References [1] and [2] shows that a fully actuated vehicle (a vehicle where the control and configuration vector have the same dimension) can be asymptotically stabilized in position and velocity by a smooth feedback law. The reasons why underactuated vehicles having zero gravitational fields are not

asymptotically stabilizable to a single equilibrium are explained in [5]. On the other hand, Wichlund K. Y et al. [6] have shown that a vehicle with gravitational and restoring terms in unactuated dynamics is stabilizable to a single equilibrium point. However, this was a necessary but not a sufficient condition to state that the vehicle is asymptotically stabilizable. The closed loop asymptotical stability of the vehicle in the earth-fixed frame needs to be examined further. In this paper, the proposed method considers the closed loop asymptotical stability and it is strongly related to feedback linearization. It cancels out all nonlinearities in the system by introducing non-linear elements in the input side; the controller design is more flexible when backstepping design methodology applied to the system [7]. The proposed controller is capable of adapting to the uncertainties of the parameters, unidentified disturbances, and noises.

The paper is organized in the following manner. A brief discussion on the dynamic modelling of AUV is presented followed by the controller design details and its stability analysis. The numerical comparative results and analysis of robustness of the controller are presented for an experimental flat fish AUV.

## II. MODELLING AND SIMULATION

The dynamic model of an underwater vehicle is developed through Newton –Euler formulation using laws of linear and angular momentum. The equations of motion of such vehicles are highly nonlinear [1] and coupled due to hydrodynamic forces which act on the vehicle. Generally, the AUV model can be described in either a body-fixed or an earth-fixed frame.

The equations of motion of an underwater vehicle in six degrees of freedom with respect to body-fixed frame can be written as [1]:

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \quad (1)$$

where,

$\mathbf{M}$  and  $\mathbf{C}(\mathbf{v})$  are the rigid body mass matrix and the Coriolis and centripetal matrix, respectively and which includes the added mass matrix and the added Coriolis and centripetal matrix respectively.  $\mathbf{D}$  is the resultant matrix of linear and quadratic drag.  $\mathbf{g}(\boldsymbol{\eta})$  is the resultant vector of gravity and buoyancy.

$\tau = [X \ Y \ Z \ K \ M \ N]^T$  is the resultant input vector of thruster, control plane forces and moments.  $v = [u \ v \ w \ p \ q \ r]^T$  is the vector of linear and angular velocities in vehicle coordinate frame.  $\eta = [x \ y \ z \ \phi \ \theta \ \psi]^T$  is the vector of absolute positions and Euler angles in Z-Y-X convention.

The relationship between linear and angular velocities in vehicle frame to those in absolute frame [refer Fig. (1)] is given by

$$\dot{\eta} = \mathbf{J}(\eta) v \quad (2)$$

where,  $\mathbf{J}(\eta)$  is the kinematic transformation matrix.

For the controller design and closed loop stability analysis, it is preferred to investigate the system from the earth fixed frame in order to maintain every state to a single reference frame. For this the coordinate transformation  $(\eta, v) \xrightarrow{\mu} (\eta, \dot{\eta})$  is performed using Eq. (2), which yields:

$$\begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}(\eta) \end{bmatrix} \begin{pmatrix} \eta \\ v \end{pmatrix} \quad (3)$$

where, the transformation matrix  $\mathbf{J}(\eta)$  is in the following form

$$\mathbf{J} = \begin{bmatrix} c\psi c\theta & -s\psi c\theta + c\psi s\theta c\phi & s\psi s\theta + c\psi c\theta c\phi & 0 & 0 & 0 \\ s\psi c\theta & c\psi c\theta + s\psi s\theta c\phi & -c\psi s\theta + s\psi c\theta c\phi & 0 & 0 & 0 \\ -s\theta & c\theta c\phi & c\theta s\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & s\phi t\theta & c\phi t\theta \\ 0 & 0 & 0 & 0 & c\phi & -s\phi \\ 0 & 0 & 0 & 0 & s\phi / c\theta & c\phi / c\theta \end{bmatrix} \quad (4)$$

where,  $s(\cdot) = \sin(\cdot)$ ,  $c(\cdot) = \cos(\cdot)$  and  $t(\cdot) = \tan(\cdot)$ . The coordinate transformation  $\mu$  is a global diffeomorphism, analogous to a similarity transformation in linear systems. This transformation is undefined for  $\theta = \pm 90^\circ$  and to overcome this singularity, a quaternion approach must be considered. However, most of the AUVs are designed to operate at pitch angles well below  $\pm 90^\circ$  and hence this limitation has no major significance here.

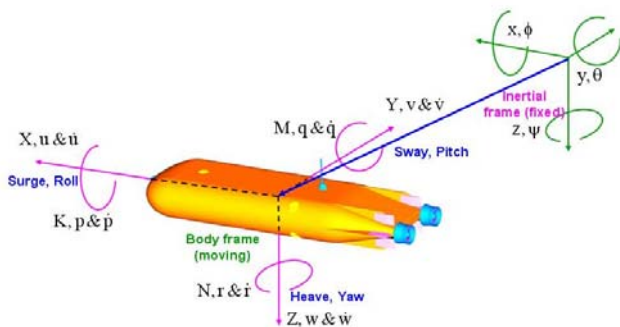


Figure 1. Body-fixed frame and earth-fixed reference frame for AUV

The AUV dynamic model in earth-fixed coordinates becomes

$$\mathbf{M}_\eta(\eta) \ddot{\eta} + \mathbf{C}_\eta(v, \eta) \dot{\eta} + \mathbf{D}_\eta(v, \eta) \dot{\eta} + g_\eta(\eta) = \mathbf{B}_\eta(\eta) u \quad (5)$$

where,

$$\mathbf{M}_\eta(\eta) = \mathbf{J}(\eta)^{-1} \mathbf{M} \mathbf{J}(\eta)^{-1}$$

$$\mathbf{C}_\eta(v, \eta) = \mathbf{J}(\eta)^{-1} (\mathbf{C}(v) - \mathbf{M} \mathbf{J}(\eta)^{-1} \dot{\mathbf{J}}(\eta)) \mathbf{J}(\eta)^{-1}$$

$$\mathbf{D}_\eta(v, \eta) = \mathbf{J}(\eta)^{-1} \mathbf{D}(v) \mathbf{J}(\eta)^{-1} \quad (6)$$

$$g_\eta(\eta) = \mathbf{J}(\eta)^{-1} g(\eta)$$

$$\mathbf{B}_\eta(\eta) = \mathbf{J}(\eta)^{-1} \mathbf{B}$$

And the above system satisfies the properties of the earth – fixed vector representation and are as follows [1]:

$$\mathbf{M}_\eta(\eta) = \mathbf{M}^T \eta(\eta) > 0, \forall \eta \in \mathcal{R}^6$$

$$s^T [\dot{\mathbf{M}}_\eta(\eta) - \mathbf{C}_\eta(v, \eta)] s = 0, \forall s \in \mathcal{R}^6, v \in \mathcal{R}^6, \eta \in \mathcal{R}^6 \quad (7)$$

$$\mathbf{D}_\eta(v, \eta) > 0, \forall v \in \mathcal{R}^6, \eta \in \mathcal{R}^6$$

### III. NONLINEAR CONTROLLER DESIGN

This section describes the design of the nonlinear tracking controller for the AUV to track a reference trajectory using the adaptive control law.

If the system is underactuated then  $\eta$  can be decomposed into the following manner

$$\eta = [\eta^a \ \eta^u]^T \quad (8)$$

where,  $\eta^a$  are actuated states and  $\eta^u$  are unactuated states of the AUV. Then the Eq. (5) can be rewritten as follows:

$$\begin{bmatrix} \tilde{\mathbf{M}}_{\eta_{11}} & \tilde{\mathbf{M}}_{\eta_{12}} \\ \tilde{\mathbf{M}}_{\eta_{21}} & \tilde{\mathbf{M}}_{\eta_{22}} \end{bmatrix} \begin{bmatrix} \ddot{\eta}^a \\ \ddot{\eta}^u \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{C}}_{\eta_{11}} & \tilde{\mathbf{C}}_{\eta_{12}} \\ \tilde{\mathbf{C}}_{\eta_{21}} & \tilde{\mathbf{C}}_{\eta_{22}} \end{bmatrix} \begin{bmatrix} \dot{\eta}^a \\ \dot{\eta}^u \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{D}}_{\eta_{11}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{D}}_{\eta_{22}} \end{bmatrix} \begin{bmatrix} \dot{\eta}^a \\ \dot{\eta}^u \end{bmatrix} + \begin{bmatrix} g_\eta^a \\ g_\eta^u \end{bmatrix} = \begin{bmatrix} \mathbf{B}_\eta^a \\ \mathbf{B}_\eta^u \end{bmatrix} u \quad (9)$$

where,  $\mathbf{B}_\eta^u = \mathbf{0}$  and  $rank(\mathbf{B}_\eta^a) = n_a =$  number of actuated states, which implies that there exists  $\mathbf{B}_\eta^{a-1} = \mathbf{B}_\eta^{a+}$  (Pseudo-inverse) or  $\mathbf{B}_\eta^{a-1}$  (inverse). Since the vehicle centre of gravity exactly lies on the origin of the body fixed coordinate frame, the cross coupled inertia terms are zero. i.e.,  $\tilde{\mathbf{M}}_{\eta_{12}} = \tilde{\mathbf{M}}_{\eta_{21}} = \mathbf{0}$ . Due to skew symmetry property, the off diagonal Coriolis and centripetal matrix terms are also zero. i.e.,  $\tilde{\mathbf{C}}_{\eta_{12}} = \tilde{\mathbf{C}}_{\eta_{21}} = \mathbf{0}$ . Therefore Eq. (8) becomes

$$\begin{bmatrix} \tilde{\mathbf{M}}_{\eta_{11}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{M}}_{\eta_{22}} \end{bmatrix} \begin{bmatrix} \ddot{\eta}^a \\ \ddot{\eta}^u \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{C}}_{\eta_{11}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{C}}_{\eta_{22}} \end{bmatrix} \begin{bmatrix} \dot{\eta}^a \\ \dot{\eta}^u \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{D}}_{\eta_{11}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{D}}_{\eta_{22}} \end{bmatrix} \begin{bmatrix} \dot{\eta}^a \\ \dot{\eta}^u \end{bmatrix} + \begin{bmatrix} g_\eta^a \\ g_\eta^u \end{bmatrix} = \begin{bmatrix} \mathbf{B}_\eta^a \\ \mathbf{B}_\eta^u \end{bmatrix} u \quad (10)$$

Considering the actuated states alone for analysis, Eq. (10) can be rewritten as follows:

$$\mathbf{M}_\eta^a \ddot{\eta}^a + \mathbf{C}_\eta^a \dot{\eta}^a + \mathbf{D}_\eta^a \dot{\eta}^a + g_\eta^a = \mathbf{B}_\eta^a u \quad (11)$$

where,  $\mathbf{M}_\eta^a = \tilde{\mathbf{M}}_{\eta_{11}}$ ,  $\mathbf{C}_\eta^a = \tilde{\mathbf{C}}_{\eta_{11}}$  &  $\mathbf{D}_\eta^a = \tilde{\mathbf{D}}_{\eta_{11}}$  are the mass/inertia, centrifugal / Coriolis and hydrodynamic damping matrices associated with the actuated dynamics, respectively.

The dynamic model in Eq. (11) that characterizes the behavior of the vehicle is in actuated state which is

composed of nonlinear functions of state variables. This feature of the dynamic model might lead us to believe that given any controller, the differential equation that models the control system in closed loop should also be composed of nonlinear functions of the corresponding state variables. This perception is confirmed for the case of most conventional control laws. Nevertheless, there exists a controller which is also nonlinear in the state variables but which leads to a closed-loop control system which is described by a linear differential equation. This proposed dynamic state feedback controller is capable of fulfilling the motion control objective, globally and moreover with a trivial selection of its design parameters.

The proposed adaptive control is given by

$$u = \mathbf{B}_\eta^a{}^{-1} \left\{ \begin{array}{l} \mathbf{M}_\eta^a (\ddot{\eta}_d^a + \mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta}) + \mathbf{C}_\eta^a \dot{\eta}^a \\ + \mathbf{D}_\eta^a \dot{\eta}^a + g_\eta^a - (\mathbf{C}_\eta^a + \mathbf{D}_\eta^a) \gamma \end{array} \right\} \quad (12)$$

$$\begin{aligned} \gamma &= -(s \tilde{\eta} + \mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta}) / (s + \lambda) \\ &= -(\ddot{\tilde{\eta}} + \mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta}) / (s + \lambda) \end{aligned} \quad (13)$$

where,  $s = d/dt$  – differential operator and  $\lambda$  is a constant which is positive value ( $\lambda > 0$ ),  $\mathbf{K}_D$  and  $\mathbf{K}_P$  are symmetric positive definite (SPD) design matrices,  $\tilde{\eta} = \eta_d^a - \eta^a$  denotes the position error,  $\dot{\tilde{\eta}} = \dot{\eta}_d^a - \dot{\eta}^a$  denotes the velocity error and  $\eta_d^a, \dot{\eta}_d^a$  &  $\ddot{\eta}_d^a$  are the desired actuated states values of positions, velocities and accelerations respectively.

Be aware that the proposed law in Eq. (12) contains the terms  $\mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta}$  which are of the PD type. However, these terms are actually pre-multiplied by the inertia matrix  $\mathbf{M}_\eta^a = \mathbf{M}(\eta_d - \tilde{\eta})$ . Therefore, this is not a linear controller as the case of PD, since the position and velocity gains are not constant but they depend explicitly on the position error  $\tilde{\eta}$ . This may be clearly seen when expressing the proposed law given by Eq. (12) as

$$u = \mathbf{B}_\eta^a{}^{-1} \left\{ \begin{array}{l} \mathbf{M}(\eta_d - \tilde{\eta}) \mathbf{K}_D \dot{\tilde{\eta}} \\ + \mathbf{M}(\eta_d - \tilde{\eta}) \mathbf{K}_P \tilde{\eta} \\ + \mathbf{M}_\eta^a \ddot{\eta}_d^a + \mathbf{C}_\eta^a \dot{\eta}^a \\ + \mathbf{D}_\eta^a \dot{\eta}^a + g_\eta^a - (\mathbf{C}_\eta^a + \mathbf{D}_\eta^a) \gamma \end{array} \right\} \quad (14)$$

The proposed control law is model-based, since it makes use of the knowledge of the matrices  $\mathbf{M}_\eta^a, \mathbf{C}_\eta^a, \mathbf{D}_\eta^a$  and the vector  $g_\eta^a$ . Besides, the desired trajectory of motion  $\eta_d^a$  and its derivatives  $\dot{\eta}_d^a$  and  $\ddot{\eta}_d^a$ , as well as the position and velocity measurements  $\eta^a$  and  $\dot{\eta}^a$  are used to compute the control action (input vector ( $u$ )) Eq. (12).

The block diagram corresponding to the proposed control method is presented in Fig. 2.

Due to the presence of the vector  $\gamma$  in Eq. (12), the control law is dynamic, that is, the control input ‘ $u$ ’ depends not

only on the actual values of the state vector formed by  $\eta^a$  and  $\dot{\eta}^a$ , but also on its past values.

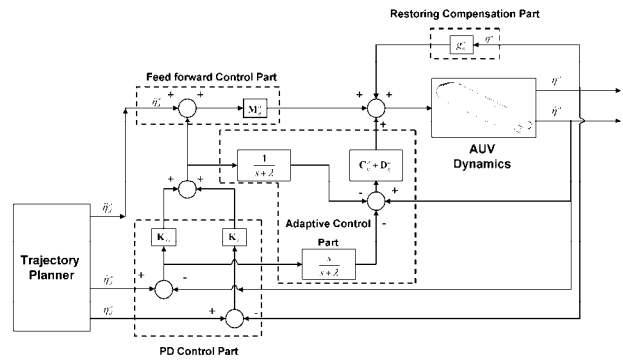


Figure 2. Proposed vehicle controller structure

This fact has a consequence that we need additional state variables to completely characterize the control law. Indeed, the expression in Eq. (12) can be written as in the state space form is a linear autonomous system given by Eq. (16)

$$\frac{d}{dt} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} -\lambda \mathbf{I} & 0 \\ 0 & -\lambda \mathbf{I} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} -\mathbf{K}_P & -\mathbf{K}_D \\ 0 & -\lambda \mathbf{I} \end{bmatrix} \begin{bmatrix} \tilde{\eta} \\ \dot{\tilde{\eta}} \end{bmatrix} \quad (15)$$

$$\gamma = \begin{bmatrix} -\mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \tilde{\eta} \\ \dot{\tilde{\eta}} \end{bmatrix} \quad (16)$$

where,  $\mu_1, \mu_2 \in \mathfrak{R}^n$  are the new state variables.

From Eqs. (11) and (12), the closed loop equation can be written as follows

$$\mathbf{M}_\eta^a (\ddot{\tilde{\eta}} + \mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta}) - (\mathbf{C}_\eta^a + \mathbf{D}_\eta^a) \gamma = 0 \quad (17)$$

Equation (17) can be rewritten in state space form using Eqs. (15) and (16) as:

$$\frac{d}{dt} \begin{bmatrix} \tilde{\eta} \\ \dot{\tilde{\eta}} \\ \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} -\mathbf{M}_\eta^a{}^{-1} (\mathbf{C}_\eta^a + \mathbf{D}_\eta^a) [\mu_1 + \mu_2 + \tilde{\eta}] - (\mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta}) \\ -\lambda \mu_1 + \mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta} \\ -\lambda \mu_1 - \lambda \tilde{\eta} \end{bmatrix} \quad (18)$$

where, the origin  $[\tilde{\eta}^T \ \dot{\tilde{\eta}}^T \ \mu_1^T \ \mu_2^T]^T = 0 \in \mathfrak{R}^{4n}$  is an equilibrium point.

To analyze the stability of the proposed control system about the equilibrium point, Eq. (13) is rewritten as:

$$\dot{\gamma} + \lambda \gamma = -(\ddot{\tilde{\eta}} + \mathbf{K}_D \dot{\tilde{\eta}} + \mathbf{K}_P \tilde{\eta}) \quad (19)$$

Substituting Eq. (19) in Eq.(17), the closed loop equation becomes:

$$\mathbf{M}_\eta^a (\dot{\gamma} + \lambda \gamma) + (\mathbf{C}_\eta^a + \mathbf{D}_\eta^a) \gamma = 0 \quad (20)$$

Consider the following Lyapunov function candidate for the closed loop equation in Eq. (18)

$$V(t, \tilde{\eta}, \gamma) = (1/2) \mathbf{M}_\eta^a \gamma^T \gamma \geq 0 \quad (21)$$

However this function not a positive definite function of the whole state, that is considering all the state variables  $\tilde{\eta}, \dot{\tilde{\eta}}, \mu_1$  and  $\mu_2$ . Since, it does not even depend on all the state variables.

The time derivative of  $V(t, \tilde{\eta}, \gamma)$  is given by

$$\dot{V}(t, \tilde{\eta}, \gamma) = \mathbf{M}_\eta^a \dot{\gamma} + (1/2) \dot{\mathbf{M}}_\eta^a \gamma, \gamma \quad (22)$$

Solving for  $\mathbf{M}_\eta^a \dot{\gamma}$  in Eq. (22) and substituting in Eq. (20), then Eq. (22) becomes

$$\dot{V}(t, \tilde{\eta}, \gamma) = -(\mathbf{M}_\eta^a + \mathbf{D}_\eta^a) \gamma, \gamma \leq 0 \quad (23)$$

where, the term  $[(1/2) \dot{\mathbf{M}}_\eta^a - \mathbf{C}_\eta^a] \gamma, \gamma$  was canceled by property which is given in Eq. (7) and similarly  $[\mathbf{M}_\eta^a + \mathbf{D}_\eta^a]$  is positive definite by property. Therefore the function in Eq. (23)  $\dot{V}(t, \tilde{\eta}, \gamma)$  is globally negative definite, i.e.,  $\dot{V}(t, \tilde{\eta}, \gamma) < 0$ . From the Lyapunov's stability theorem, that  $\gamma$  is converges zero in exponential manner, which can be written as

$$\lim_{t \rightarrow \infty} \gamma(t) = 0 \quad (24)$$

On the other hand, the Eq. (13) can be written as

$$\gamma = -(s^2 \mathbf{I} + s \mathbf{K}_D + \mathbf{K}_P) \tilde{\eta} / (s + \lambda) \quad (25)$$

In other form, the Eq. (25) becomes

$$\tilde{\eta} = -(s^2 \mathbf{I} + s \mathbf{K}_D + \mathbf{K}_P)^{-1} (s + \lambda) \gamma \quad (26)$$

Since  $\lambda > 0$ , while  $\mathbf{K}_D$  and  $\mathbf{K}_P$  are symmetric positive definite matrices. Therefore from Eq. (26), the input to this system is  $\gamma$  which tends to zero exponentially fast, and its output  $\tilde{\eta}$ . So the fact is that a stable strictly proper filter with an exponentially decaying input produces an exponentially decaying output, that is,

$$\lim_{t \rightarrow \infty} \tilde{\eta}(t) = 0 \quad (27)$$

Which means the proposed control system closed loop equation is asymptotically stable.

#### IV. NUMERICAL SIMULATIONS

The aim of this numerical simulation is to illustrate the performance of the proposed controller on an experimental autonomous underwater robot which is being developed at the Indian Institute of Technology Madras [8]. The vehicle is an under-actuated AUV; it has three control inputs, namely a set of thrusters to control surge motion (refer Fig.1), and rudder surface and lateral moving surfaces (stern) to control yaw and pitch motion respectively [8, 9]. Surge speed, pitch, and yaw are considered here as controlled variables and thrust, stern plane angle, and rudder angle as the control variables (roll is not considered here as it cannot be controlled in this case).

This experimental vehicle is positive buoyant and has three control inputs, namely, propulsion thrusters for surge control and rudder and stern plane for attitude control. The robustness and the performance of the model based controller are compared with classical PID control to illustrate the significance and performance improvement of the former [9]. For the system with governing equations as in Eq. (11) can be written as:

$$u = \mathbf{B}_\eta^{-1} \{ \mathbf{K}_p e + \mathbf{K}_D \dot{e} + \mathbf{K}_I \int e dt \} \quad (28)$$

Simulation of the vehicle is performed using fourth order Runge - Kutta method with a fixed step size of 0.02s. The controller parameters (PID gains) are tuned by

considering the actuator characteristics such as actuator dynamics (response time), limits of actuators (saturation), etc using Taguchi robust design method.

The desired trajectory as a function of time is depicted in Fig. 3 which is derived from trajectory planner with the help of via points. The vehicle accelerates for the initial 50s and reaches a constant speed of 1.5m/s

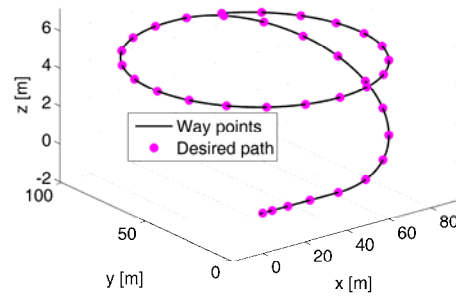


Figure 3. Desired 3D path with way points

The comparative results of proposed adaptive control and PID control for trajectory tracking are given in Figs.4-6, and the results show that the performance of proposed technique is better than the conventional PID control.

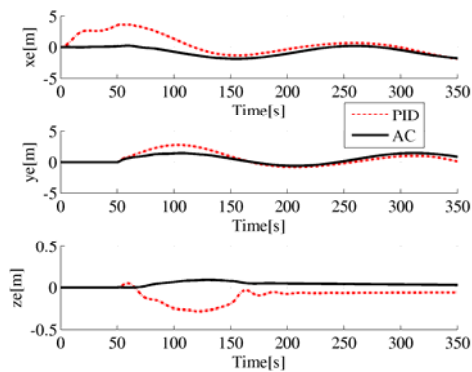


Figure 4. Position tracking errors of both PID & Adaptive control(AC)

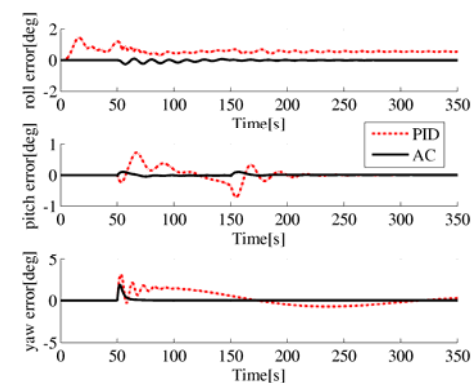


Figure 5. Attitude tracking errors

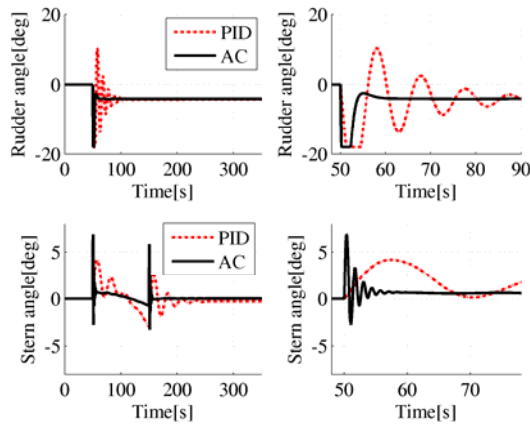


Figure 6. Control plane inputs (a)&(c) full view, (b)&(d) zoomed view

In order to demonstrate the adaptability of the proposed controller, an error of 10%, 20% and 50% in vehicle and hydrodynamic parameters is introduced in the system. Simulations were carried out for the trajectory tracking for the given 3D trajectory (refer Fig.3). The results are presented in Figs. 7-9. These results show that the controller successfully adapts the parameter variations to a great extent.

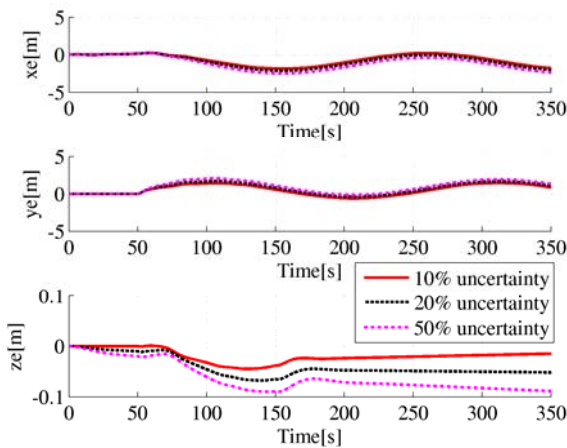


Figure 7. Position tracking errors for parameter variations

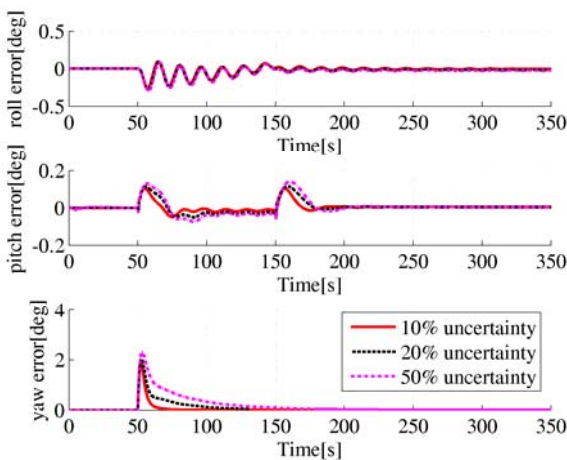


Figure 8. Attitude tracking errors for parameter variations

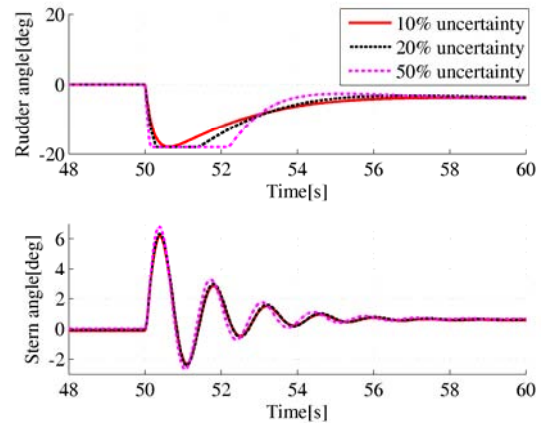


Figure 9. Control plane (actuator) inputs for parameter variations

V. CONCLUSIONS

This paper presents a new adaptive nonlinear control strategy for controlling an AUV without considering any confining assumptions. The proposed control system is developed by dynamic state feedback adaptive control scheme. The control algorithm as well as simulation results are presented. Simulation results show that the controller is quite efficient in reducing the performance error and system exhibits better stability characteristics for path control applications. Further work on analyzing the controller performance for various operational situations like underwater current, external disturbance, sensor noise etc. is under progress.

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