FULL PAPER
International Journal of Recent Trends in Engineering, Vol 2, No. 5, November 2009

On-Set Theory of Self-Excitation in Induction Generator

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Abstract--**This paper examines the phenomenon of self-excitation in an induction generator which is of practical interest. Therefore the advanced knowledge of the minimum excitation capacitor value is required .To find this capacitor value two non-linear equations have to be solved. Different numerical methods for solving these equations are known from previous literature. However, these solutions involve some guessing in a trial-and-error procedure. In this paper, a new simple and direct method is developed to find the capacitance requirement under R-L load. Exact values are derived for the minimum capacitance required for self-excitation and the output frequencies under no-load, inductive load and resistive load. These calculated values can be used to predict theoretically the minimum value of terminal capacitance required for self-excitation. For stable operation C must be chosen to be slightly greater than Cmin. Furthermore, it is found that there is a speed threshold below which no excitation is possible what the capacitor value. This threshold is called cut off speed. Expressions for this speed under no load and inductive load are also given.**

Index Terms: **Capacitance requirements, self-excitation, induction generator**

I. INTRODUCTION

The principle of self-excitation, by which an induction generator can be excited from static capacitors is well known [1, 2]. The utilization of such an idea in the generation of electrical power was realized in recent years and growing interest in the use of other energy sources. This has been motivated by concern to reduce pollution by the use of renewable energy resources such as wind, solar, tidal and small hydro potential .Owing to its many advantages, the self-excited induction generator has emerged from among the known generators as a suitable candidate to be driven by wind power. Some of its advantages are small size and weight, robust construction, absence of separate source of excitation and reduced unit and maintenance cost [1, 3-5].Besides its application as a generator, the principle of self-excitation can be used in dynamic braking of a three-phase induction motor [6, 7]. Therefore, methods to analyze the performance of such machines are of considerable practical interest. The terminal capacitance on such machines must have a minimum value so that self-excitation is possible. This value is affected by machine parameters, its speed and load condition. Over the past decade, many researchers have attempted to analyze the SEIG. Using the equivalent circuit approach, Grantham et. al analyzed the process of voltage build up in the SEIG using generalized machine theory and proposed the onset theory of self-excitation [8]. Earlier, Malik and Mazi investigated the capacitance requirements of the SEIG using a trial and error method based on the steady –state equivalent circuit model [9]. Alolah et. al attempted a direct method of computing C_{min}, but the work seemed to have been devoted to the derivation of closed form solutions for certain special cases of load impedance [10]. Experimental results presented by all the above researches referred to the no-load case only. T.F. Chan presented a simple method for computing the minimum value of capacitance required for initiating voltage build up in a three-phase self-excited induction generator [11].Several papers have investigated the capacitance requirements of the SEIG [8-14]. It has practical significance as it enables the design and operation engineer to select the proper value of excitation capacitance for specific machine.

This paper introduces a new simple and direct method of finding the minimum capacitance required for self-excitation. The exact values for the minimum capacitance under no-load, inductive and resistive loads are derived. Furthermore, it is shown that there is a speed threshold, below which no excitation is possible no matter what the capacitor value.

II. CAPACITOR SELF-EXCITATION, ANALYSIS

Fig1. (a) shows the equivalent circuit commonly used for the steady-state analysis of the SEIG [15]. For the machine to self-excite, the excitation capacitance must be larger than some minimum value. In order to obtain a stable output voltage, the machine must operate at an appreciable level of magnetic saturation. Accordingly, the magnetizing reactance X_m is not constant, but varies with the load and circuit conditions. For successful voltage build-up, the load-capacitance combination should result in a value of X_m which is less than the unsaturated value, hence the condition $X_m = X_{max}$ yields the minimum value of excitation capacitance below which the SEIG fails to self-excite.

There are two different approaches in the steady-state analysis of self-excited induction generators. They are the loop impedance method as used by Malik et al. [16, 9] and the nodal admittance method as used by Mcpherson et al. [17, 18]. If the machine speed is specified and the condition $X_m = X_{max}$ prevails, then the only variables in the equivalent circuit of Fig.1 (a) are the per- unit frequency f and the capacitive reactance X_c.

The nodal admittance method will be used instead, the advantage being that the load and excitation capacitance branches can be easily decoupled, which enables the per unit frequency f to be determined independent of the value of X_c.

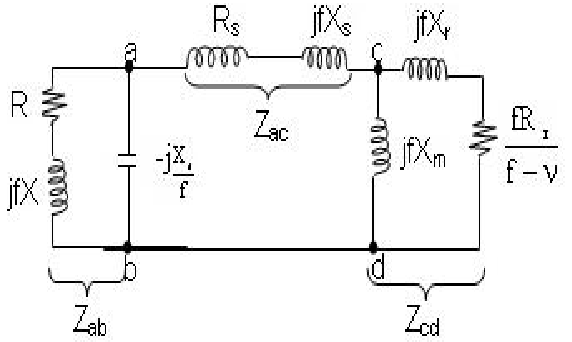

Figure1 (a). Equivalent circuit of SEIG

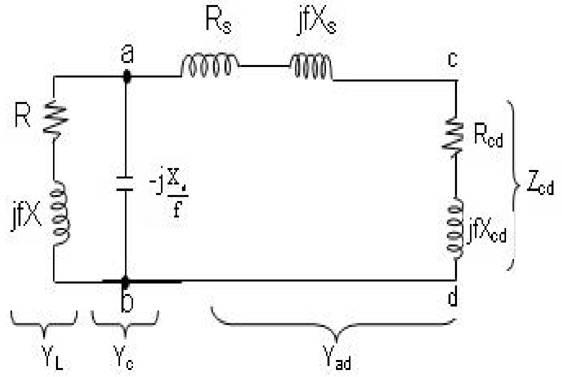

Figure1 (b). Simplified Equivalent circuit

For this purpose, Fig1 (a) is redrawn as Fig1 (b), where

$$Z_{cd} = \left[\frac{fRr}{(f-v)} + jf\,X_r\right] // jf\,X_m \qquad (1)$$

Separating real and imaginary Parts

$$R_{cd} = \frac{f(f-v)R_r\,X_m^2}{R_r^2 + (f-v)^2(X_r + X_m)^2} \qquad (2)$$

$$X_{cd} = \frac{f\left[R_r^2\ X_m + (f-v)^2.X_r\,X_m\ (X_r + X_m)\right]}{R_r^2 + (f-v)^2(X_r + X_m)^2} \qquad (3)$$

The total impedance Z_{ad} of branch acd is then given by

$$Z_{ad} = R_{ad} + jX_{ad} \qquad (4)$$

$$R_{ad} = R_s + R_{cd} \qquad (5)$$

$$X_{ad} = fX_s + fX_{cd} \qquad (6)$$

$$Z_L = R + jfX$$

The admittances Y_L and Y_{ad} are given by

$$Y_L = \frac{R}{R^2 + f^2X^2} - j\frac{fX}{R^2 + f^2X^2} \qquad (7)$$

$$Y_{ad} = \frac{R_{ad}}{R_{ad}^2 + X_{ad}^2} - j\frac{X_{ad}}{R_{ad}^2 + X_{ad}^2} \qquad (8)$$

By Kirchoff's Law, the sum of currents at node a should be equal to zero, whence

$$\frac{V_1(Y_c + Y_L + Y_{ad})}{a} = 0 \qquad (9)$$

For successful voltage build-up, V1≠0, hence

$$Y_c + Y_L + Y_{ad} = 0 \qquad (10)$$

Equating the real and imaginary parts to zero

$$\frac{R}{R^2 + f^2X^2} + \frac{R_{ad}}{R_{ad}^2 + X_{ad}^2} = 0 \qquad (11)$$

$$\frac{f}{X_C} - \frac{fX}{R^2 + f^2X^2} - \frac{X_{ad}}{R_{ad}^2 + X_{ad}^2} = 0 \qquad (12)$$

It is noted that (11) is independent of X_c & the only variable is the per unit frequency f. Once the value of f has been determined then X_c can be determined using (12).

Equation (11), after a series of algebraic manipulation can be expressed as a 6th degree polynomial in f as

$$P_6f^6 + P_5f^5 + P_4f^4 + P_3f^3 + P_2f^2 + P_1f + P_0 = 0 \qquad (13)$$

The derivation of these constants (coefficients) P_0 to P_6 is given in Appendix-A. Equation (13) can be solved numerically to yield all the real and complex roots. Only the real roots have physical significance and the largest positive real root yields the per – unit frequency that corresponds to C_{min} i.e.

$$f_{max} = \max\{f_i, i \le 6\} \qquad (14)$$

Where $\{f_i, i \le 6\}$ is the set of positive real roots of (13).

Having determined f_{max}, equation (12) may be used to calculate C_{min} as follows:

$$C_{min} = \frac{1}{2\pi f_b Z_b f_{max}}\left[\frac{f_{max}X}{R^2 + f_{max}^2X^2} + \frac{X_{ad}}{R_{ad}^2 + X_{ad}^2}\right] \qquad (15)$$

A. SPECIAL CASES

The proposed method can be used to predict C_{min} for all types of connected load impedance by putting suitable values of R & X in (7). However, for no load [9-11] and pure inductive loads [10, 11] closed form solution exists for the self excited frequency and C_{min}. Also, an analytical expression can be derived for the critical speed v_c [9, 10].

For no – load operation

$$R = \infty \text{ and } X = 0$$

The circuit of fig.1 becomes

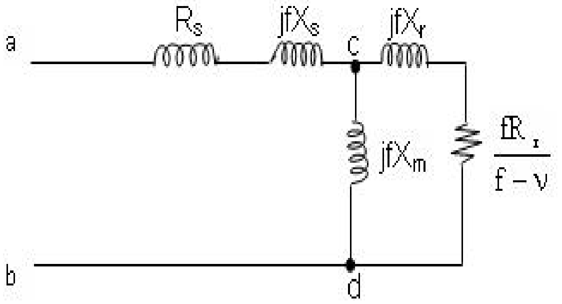

Figure 2. Equivalent circuit under no-load

Substituting R = ∞ and X = 0 in (11)

$$R_s + \frac{f(f-v)R_r X_m^{\ 2}}{R_r^{\ 2} + (f-v)^2 (X_r + X_m)^2} = 0 \qquad (16)$$

On simplification, it yields the following.

$$f_{\max} = v - \frac{v}{2}\left\{ \frac{1 - \sqrt{1 - \left(\frac{v_c}{v}\right)^2}}{1 + \frac{R_s}{R_r}\left(1 + \frac{X_r}{X_m}\right)^2} \right\} \qquad (17)$$

Where v_c is given by

$$v_c = \frac{2R_s}{X_m}\sqrt{\frac{R_r}{R_s} + \left(1 + \frac{X_r}{X_m}\right)^2} \qquad (18)$$

Substituting R = ∞ and X = 0 in (12)

$$X_c = f_{\max}^2 (X_s + X_{cd}) \qquad (19)$$

Hence C_{min} is given by

$$C_{\min} = \frac{1}{\left\{2\pi f_b Z_b f_{\max}^2 (X_s + X_{cd})\right\}} \qquad (20)$$

Thus, C_{min} is inversely proportional to the square of the p.u. machine frequency (or machine speed v). Further more, it is also inversely proportional to the unsaturated magnetizing reactance X_m (Since $X_{cd} = X_m$ when f = v, also $X_s + X_m \approx X_m$ = unsaturated magnetizing reactance).

The value of C_{min} determined from (20) is just sufficient to have self-excitation under steady state. If a terminal capacitor C = C_{min} is used and the generator is started from rest, the voltage build up will not take place. Thus in practice, terminal capacitor C having a value somewhat greater than C_{min} should be selected to ensure self-excitation.

For pure inductive loads

Equation (16) is again obtained upon equating the real terms in (11) to zero. Hence, the self-excited frequency is also given by (17).

C_{min} can then be expressed as

$$C_{\min} = \frac{1}{2\pi\pi_b z_b f_{\max}}\left\{\frac{X + fX_s + X_{cd}}{X(fX_s + X_{cd})}\right\} \qquad (21)$$

For resistive load

Substitute X = 0 in equations (A-1) to (A-12). The coefficients of the sixth order polynomial in (13) get modified and are given in appendix B. It is noted from appendix-B that for the resistive load only, the coefficients P_0 and P_1 of the sixth order polynomial remains unchanged while the other coefficients gets modified but the order of the polynomial also remained the same.

III.COMPUTER ALGORITHM

In order, to develop a computer algorithm to determine C_{min} for self – excitation of SEIG using the techniques described in section – it is desirable to have a program or subroutine to calculate the roots of a polynomial with complex coefficients and also to fit a curve showing the variation of X_m with the air-gap flux. This curve has to be fitted using the observations from the 'synchronous speed' test. The flowchart of the computer programme is given in flowchart

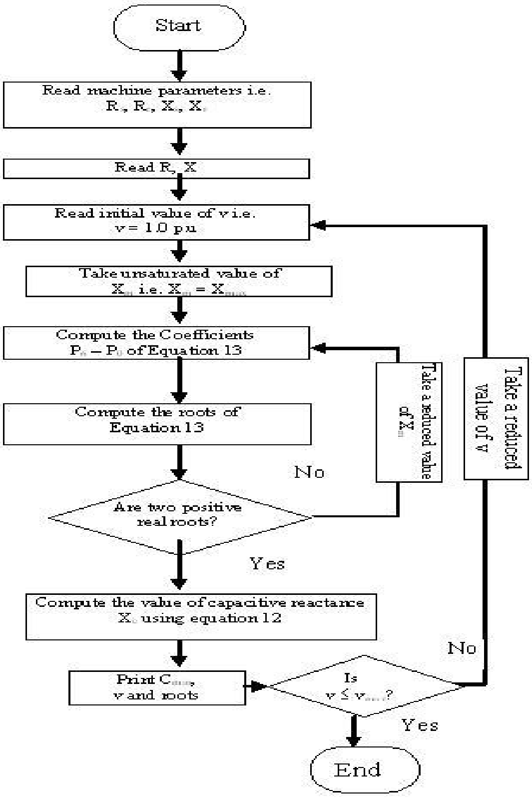

Figure3. Flowchart to determine Cmin

IV.COMPUTED RESULTS AND DISCUSSION

In this paper, the computed results are obtained by the procedures and calculations outlined above, number of experiments were conducted using a 3-ϕ induction machine coupled with a D.C. shunt motor. The induction machine was a three-phase, 400/440V, 8.5A, 50Hz, 4.5/6 kW/hp, 4-pole, 1440 rpm, star connected stator winding. The machine was coupled to a D.C. shunt motor to provide different constant speeds. A three-phase variable capacitor bank or a single capacitor was connected to the machine terminals to obtain self-excited Induction generator action. The measured machine parameters were: R_S = 0.068993 p.u , R_r = 0.012492 p.u. **,** $X_s = X_r$ = 0.074575 p.u., X_m = 2.157066 p.u .As an example, consider the case when the machine is driven at rated speed with a connected load impedance of (0.8 + j0.6) i.e.

R = 0.8 p.u, X = 0.6 p.u. And v = 1.0 p.u

From (13)

$$P_6f^6+ P_5f^5 + P_4f^4 + P_3f^3 + P_2f^2 + P_1f + P_0 = 0$$

P_6 ----- P_0 are constants whose numerical values are obtained by using MATLAB Software Package.

The following polynomial is obtained upon numerical substitution.

$$0.7325f^6–2.8639f^5+5.0979f^4-6.1949f^3+5.6801f^2–3.1838f+0.7325=0 \qquad (22)$$

Solution of (22) yielded the following complex and positive real roots.

f_1 = 0.0177 + j 1.1012 p.u
f_2 = 0.0177 + j 1.1012 p.u
f_3 = 1.1379 + j 0.2734 p.u
f_4 = 1.1379 + j 0.2734 p.u
f_5 = 0.9910 p.u (= 49.55 Hz)
f_6 = 0.6074 p.u (= 30.37 Hz)

As only the real roots have physical significance and the largest real root yields the per – unit frequency f_{max} that corresponds to C_{min} i.e.

$$f_{max} = \max \{f_i, i < 6\}$$

Where $(f_i, i < 6\}$ is the set of positive real roots of (13).

Let $(C_i, i < 6\}$ be the corresponding set of positive capacitor values. Therefore the two values of C_{min} corresponding to two positive real roots are.

f = 0.9910
f = 0.6074

Since all these values & C are sufficient to guarantee self – excitation of the induction generator, it follows that the minimum capacitor value required is given by C_{min}. It is seen that only the larger positive real root gives the feasible value of C_{min}. The smaller real root on the other hand gives the value of excitation capacitance above which the machine fails to excite. However, such a condition is unpractical as the corresponding excitation current would far exceed the rated current of the machine.

If (22) has no real roots, then no excitation is possible. Also, there is a minimum speed value, below which (22) have no real roots. Correspondingly, no excitation is possible.

It is noted that for R-L loads, there are in general two real roots and two pair of complex conjugate roots. This restricts the set of two capacitor values. It is also noted that f < v i.e. the p.u slip s = f – v is always negative as it should be for generator operation. If f > v then from fig.1 R_r.(f – v) is strictly positive and therefore no excitation is possible.

No load Capacitance Requirements

As for the no load case closed form solutions exist for the self – excitation frequency f_{max} and C_{min}. [(17) and (18)]. Also an analytical expression was also derived for the critical speed v_c (18). The critical speed v_c is the speed below which the machine will not operate. For the given machine parameters i.e. R_S, R_r, X_S, X_r, assuming $X_S = X_r$ [15], speed v and magnetizing reactance $X_m = X_{max}$, Equation (17) was solved to obtain the p.u. frequency f_{max} corresponding to self – excitation and the critical speed v_c was obtained from (18), for each value of p.u speed, the freq f_{max} was be calculated. Table 1. shows the variation of f_{max} for different p.u speeds v with X_m=2.157066.

TABLE 1.

Variation of f_{max} with speed
Base speed = 1500 rpm

Speed (p.u)	**F $_{max}$ (p.u)**
1.2	1.1998
1	0.9998
0.9	0.89979
0.8	0.79976
0.7	0.6997
0.6	0.5997
0.5	0.4996
0.4	0.3995
0.3	0.2994
0.2	0.1990
0.1	0.0978

It is noted from table 2. that when conditions for self – excitation are just fulfilled ($C = C_{min}$), f_{max} is very nearly equal to the p.u speed v.

Having determined f_{max} from (17), X_c can be determined from (19). It was noted from equation (17) that X_c is an increasing function of f_{max}. Then minimum value of capacitance required for self excitation was obtained from (20). It was noted from (20) that C_{min} increases with decrease in f_{max}.

Inductive load capacitive Requirements-

For Inductive load also, the closed form solutions were obtained for self– excitation frequency f_{max} and minimum capacitance required for excitation of SEIG i.e. C_{min}. The self-excitation frequency and the critical speed for the inductive load was same as for the no load case [19 & 18]. These approximation for v_c and f_{max} here is valid only if the condition of approximation is satisfied i.e. the speed must be much greater than the cut off speed i.e. $v > v_c$; For the given machine parameters & inductive load, the value of C_{min} was obtained from (21).

CONCLUSIONS

A computer oriented program has been developed to find the minimum capacitance required for a capacitor self-excited induction generator. These values can be used to predict theoretically the minimum values of terminal capacitance required for self-excitation. Of course, for a stable operation of the machine C must be chosen slightly greater than C_{min}. Exact expressions for capacitor values under no-load, inductive load and resistive load s and the corresponding output frequencies are also derived. The theoretical results of no-load derived here show a good agreement with the experimental measurements carried out previously [9, 10 and 13].

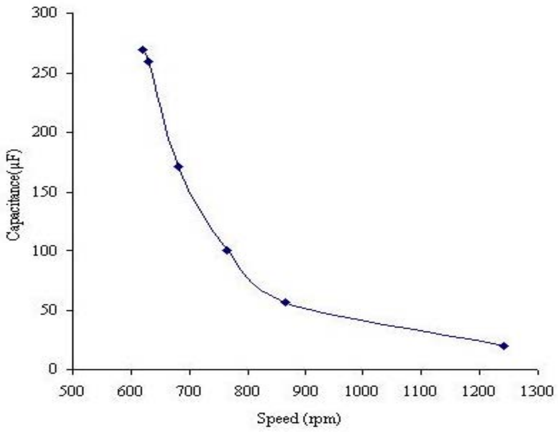

Figure 4. Variation of Cmin with speed for the balanced excitation under no load

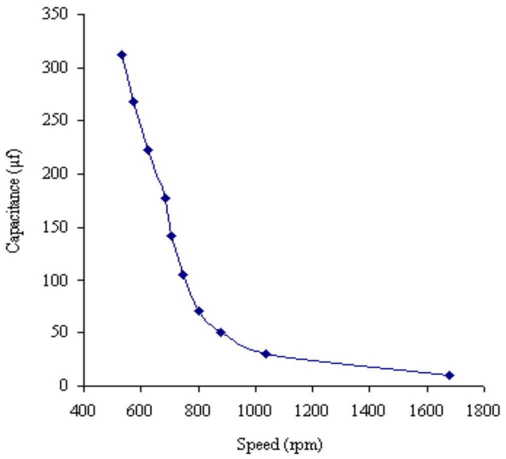

Figure. 5 Variation of Cmin with speed for the unbalanced excitation under no load

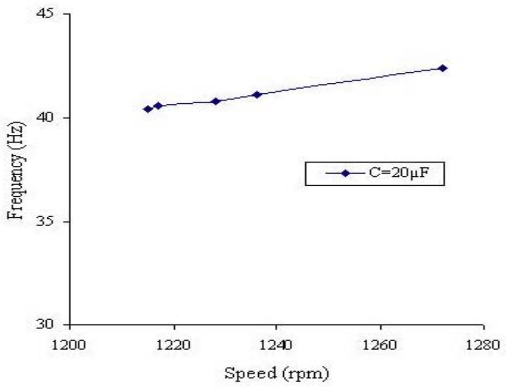

Figure. 6. Variation of frequency with speed under no load

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APPENDIX A

To compute the coefficients P_0 and P_6 in (9), the following are first defined:

$$X_3 = X_m + X_r \qquad \text{(A-1)}$$

$$DENOM = R_r^{\,2} + (f - v)X_3^{\,2} \qquad \text{(A-2)}$$

$$NUM1 = R_{s.}DENOM f\,(f - v)R_r\,Xm^2 \qquad \text{(A-3)}$$

$$NUM2 = f\,[X_{s.}\,DENOM + X_m\,\{R_r^{\,2} + (f - v)^2 X_r X_3\}] \qquad \text{(A-4)}$$

$$Z_{ad} = R_{ad} + jX_{ad}$$

$$= \frac{NUM1 + j\,NUM2}{DENOM} \qquad \text{(A-5)}$$

Equating (7), upon cross-multiplication, becomes

$$R(NUM1^2 + NUM2^2) + (R^2 + f^{\,2}X^2)DENOM.NUM1$$
$$= 0 \qquad \text{(A-6)}$$

DENOM, NUMI and NUM2 can be reduced to the following forms:

$$DENOM = g_2 f^{\,2} + g_1 f + g_0 \qquad \text{(A-7)}$$

$$NUM1 = h_2 f^{\,2} + h_1 f + h_0 \qquad \text{(A-8)}$$

$$NUM2 = k_3 f_2 + k_1 f^{\,2} + k_1 f \qquad \text{(A-9)}$$

Where

$$\left.\begin{aligned} g_0 &= R_r^{\,2} + v^2 X_3 \\ g_1 &= -\,2vX_3^{\,2} \\ g_2 &= X_3^{\,2} \end{aligned}\right\} \qquad \text{(A-10)}$$

$$\left.\begin{aligned} h_0 &= R_s\,(R_r^{\,2} + v^2\,X_3^{\,2}) \\ h_1 &= -\,v\,(R_r X_m^{\,2} + 2R_s X_3^{\,2} \\ h_2 &= R_r X_m^{\,2} + R_s X_3^{\,2} \end{aligned}\right\} \qquad \text{(A-11)}$$

$$\left.\begin{aligned} k_1 &= (X_s R_r^{\,2} + X_m R_r^{\,2}) + v^2\,(X_s X_3^{\,2} + X_r X_3 X_m) \\ k_2 &= -\,2v\,(X_s X_3^{\,2} + X_r X_3 X_m) = -\,2vk_3 \\ k_3 &= X_s X_3^{\,2} + X_r X_3 X_m \end{aligned}\right\} \qquad \text{(A-12)}$$

Each of the terms in (7), after expansion reduces to a 6th degree polynomial, whose coefficients P_0 to P_6 are given below:

$$P_6 = k_3^2 R + g_2 h_2 X^2$$

$$P_5 = g_2 h_1 X^2 + g_1 h_2 X^2 + 2k_2 k_3 R$$

$$P_4 = (h_2^{\,2} + k_2^{\,2} + 2k_1 k_3)R + (g_2 h_0 + g_1 h_1 + g_0 h_2)X^2 + R^2 g_2 h_2 \qquad \text{(A-1)}$$

$$P_2 = (g_2 h_0 + g_1 h_1 + g_0 h_2)R^2 + g_0 h_0 X^2 + (h_1^{\,2} + k_1^{\,2} + 2h_0 h_2)R$$

$$P_1 = (g_1 h_0 + g_0 h_1)\,R^2 + 2h_0 h_1 R$$

$$P_0 = h_0^{\,2} R + g_0 h_0 R^2$$

The coefficients P_0 to P_6 are systematically expressed in terms of R, X and the constants defined in (A-10) to (A-12).

APPENDIX B

For resistive load

Substitute X = 0 in (A-1) to (A-12).The modified coefficients are as follows:

$$P_6 = k_3^2 R$$

$$P_5 = 2k_2 k_3 R$$

$$P_4 = (h_2^{\,2} + k_2^{\,2} + 2k_1 k_3)R + R^2 g_2 h_2$$

$$P_3 = (g_2 h_1 + g_1 h_2)R^2 + (2h_1 h_2 + 2k_1 k_2)R$$

$$P_2 = (g_2 h_o + g_1 h_1 + g_o h_2)R^2 + (h_1^{\,2} + k_1^{\,2} + 2h_o h_2)R$$

$$P_1 = (g_1 h_o + g_o h_1)R^2 + 2h_o h_1 R$$

$$P_o = h_o^{\,2} R + g_o h_o R^2$$