

# Design of Decentralized Fuzzy Pre compensated PI Controllers for Quadruple Tank System

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**Abstract—** The Quadruple Tank system has been used to illustrate both traditional and advanced multivariable control strategies and can be utilized as an educational tool in teaching advanced multivariable control techniques and provide interesting challenges in the control design. The main feature of the Quadruple Tank System is the flexibility in positioning one of its multivariable zeros on either half of the 's' plane. The objective of the current study is to design a Decentralized Fuzzy pre compensated PI controllers for a multivariable laboratory Quadruple Tank System. Simulation results confirm the effectiveness of the proposed control methodology

**Index Terms—** Fuzzy control, Quadruple Tank System, Decentralized control, non-minimum phase system

## I. INTRODUCTION

Chemical Plants are tightly integrated processes, that exhibit non-linear behavior and complex dynamic properties. Chemical manufacturing processes present many challenging control problems due to their non-linear dynamic behaviour, uncertain and time-varying parameters, constraints on manipulated and state variables, multivariable interactions between manipulated and controlled variables, unmeasured state variables, unmeasured disturbances, high order and distributed processes, dead time on input and measurements. In particular, it is well recognised that one of the most important characteristics of chemical processes that present formidable control problem is the inherent non-linearity of the process.

The Quadruple Tank System has been used in control literature to illustrate many concepts in multivariable control; particularly performance limitations due to multivariable right half plane zeros. It exhibits elegantly complex dynamics, which emerge from a simple cascade of tanks. Such dynamic characteristics include interactions and transmission zero location that are tunable in operation. With appropriate tuning, this system exhibits non-minimum phase characteristics that arise completely from the multivariable nature of the problem. The linearized dynamics of the system have multivariable zero that is possible to move along the real axis by changing the valve. The zero can be placed in both the left and right half of s plane. The location and the direction of the zero have an appealing physical interpretation. To

control a quadruple tank system, one essential problem is how to handle the interactions among two loops. An effective approach is to apply the so called decentralized control strategy: each loop is controlled by one controller independently based on local information and local actions.

Fuzzy control has found promising applications for a wide variety of industrial systems based on the universal approximation capability, many effective fuzzy control schemes have been developed to incorporate with human experts knowledge and information in a systematic way, which can also guarantee various stability and performance criteria, not only for SISO nonlinear systems but also for MIMO nonlinear systems. The main advantages of these fuzzy-logic-based control schemes lies in the fact that the developed controllers can deal with increasingly complex systems and to implement controllers without precise knowledge of the model structure of underlying dynamic systems.

This paper proposes a pre compensated fuzzy logic concept to improve the performances of existing decentralized PI control systems. The main advantage of using pre compensated fuzzy logic control structure is that one does not have to redesign the existing control system but also acquire the satisfactory response when disturbances and noises enter.

The rest of the paper is structured as follows: Section 2 describes the Quadruple Tank System. The design of controllers for Quadruple Tank System is explained in Section 3 followed by Results and discussions in Section 4. The conclusion is explained in section 5.

## II. QUADRUPLE TANK SYSTEM

This is a new laboratory setup, which was designed to illustrate performance limitations due to zero location in multivariable control systems. The Quadruple Tank System consists of four interconnected water tanks and two pumps. The system is shown in Figure.1. Its manipulated variables are voltages to the pumps and the controlled variables are the water levels in the two lower tanks. The quadruple-tank system can easily be built by using two double-tank processes. The output of each pump is split into two using a three-way valve. Thus each pump output goes to two tanks, one lower and another

upper, diagonally opposite and the ratio of the split up is controlled by the position of the valve. With the change in position of the two valves, the system can be appropriately placed either [1] in the minimum-phase or in the non-minimum phase. The physical parameters of the process given by Johansson [1] are given in Table 1. The material balance for the quadruple-tank system is given by the Equations 1 to 4. Note that  $\gamma_1$  and  $\gamma_2$  are the ratios in which the outputs of the two pumps get divided. If  $\gamma_1$  is the ratio of flow to the first tank, then  $1 - \gamma_1$  will be the flow to the fourth tank. As the inputs to the pumps are the voltages  $v_1$  and  $v_2$ ,  $k_1$  and  $k_2$  are conversion factors, expressed in flow per unit voltage input to the pumps.

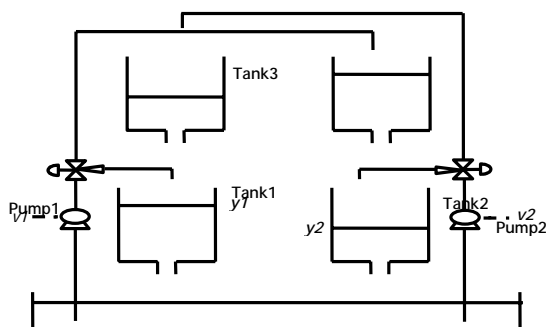


Figure 1. Quadruple Tank System

The following equations represent the mathematical model of the Quadruple Tank System

$$\begin{aligned} \frac{dh_1}{dt} &= -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_1} \sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1 \\ \frac{dh_2}{dt} &= -\frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_4}{A_2} \sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2 \quad (1) \\ \frac{dh_3}{dt} &= -\frac{a_3}{A_3} \sqrt{2gh_3} + \frac{(1 - \gamma_2) k_2}{A_3} v_2 \\ \frac{dh_4}{dt} &= -\frac{a_4}{A_4} \sqrt{2gh_4} + \frac{(1 - \gamma_1) k_1}{A_4} v_1 \end{aligned}$$

Where,  $A_i$  -Cross sectional area of the tank ( $m^2$ ),  $a_i$  -Cross sectional area of the outlet hole ( $m^2$ ),  $h_i$  - Water level (m). The parameter values of the quadruple tank System are represented in the Table 1.

TABLE 1  
PHYSICAL PARAMETERS OF QUADRUPLE TANK SYSTEM

Sl.no	Description	Value
1	Area of the tanks $A_1 A_3$	28 $cm^2$
2	Area of the tanks $A_2 A_4$	32 $cm^2$
3	Area of outlet pipes $a_1 a_3$	0.071 $cm^2$
4	Area of outlet pipes $a_2 a_4$	0.057 $cm^2$
5	Constant $k$	0.50 V/cm
6	Gravitational constant $g$	981 $cm/s^2$

The model and control of the process are studied at two operating points;  $P_-$  at which the system exhibits minimum-phase characteristics and  $P_+$  at which the system exhibits non-minimum characteristics. The chosen operating points correspond to the values given in the Table 2.

TABLE 2  
OPERATING PARAMETERS OF MINIMUM-PHASE AND NON-MINIMUM-PHASE SYSTEM

Parameters	Minimum Phase	Non Minimum Phase
$h_1^0, h_2^0$	12.4, 12.7	12.6, 13.0
$h_3^0, h_4^0$	1.8, 1.4	4.8, 4.9
$v_1^0, v_2^0$	3.00, 3.00	3.15, 3.15
$k_1, k_2$	3.33, 3.35	3.14, 3.29
$\gamma_1, \gamma_2$	0.70, 0.60	0.43, 0.34

## II. DESIGN OF CONTROLLERS

### A. Decentralized PI Controller

The control objective for Quadruple Tank System is to reach the given level in the lower two tanks, i.e. Prescribed values of  $y_1$  and  $y_2$  by controlling input flow  $v_1$  and  $v_2$  delivered by two pumps. In order to achieve this objective, the decentralized control structure is employed, with two control loops with respect to output values  $y_1$  and  $y_2$ . Therefore the appropriate input-output pairing has to be chosen for both configurations using steady-state RGA index. The control structure for minimum phase shown in Figure 2 has individual PI controller for individual loops. The manipulated variables are the function of error of that particular loop. The pairing of the loops is decided by the Relative Gain Array (RGA) analysis

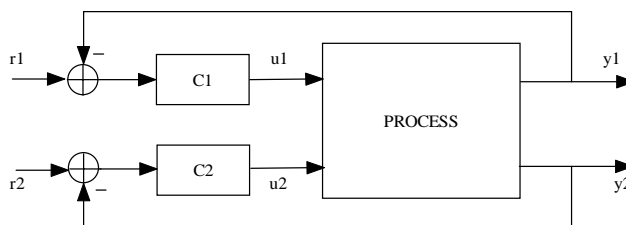


Figure 2. Multi-loop control structure for minimum-phase system with two PI controllers C1 and C2.

For minimum-phase settings,  $\lambda$  is 1.40. So,  $u_1$  must be paired with  $y_1$  and  $u_2$  must be paired with  $y_2$  for better performance. For non-minimum phase system,  $\lambda$  is 0.64. So,  $u_1$  must be paired with  $y_2$  and  $u_2$  must be paired with  $y_1$  to achieve good control performance.

The PI controller equation is given by

$$Y_j(s) = K_j (1 + 1/T_{ij} s) \quad , j=1,2. \quad (2)$$

The controller parameters are obtained by the direct synthesis method both for the minimum phase system and non-minimum phase system.

For the minimum-phase the controller settings  $\{K_1, T_{i1}\} = \{2.39, 62\}$  and  $\{K_2, T_{i2}\} = \{3.21, 90\}$  gives better performance. The non-minimum-phase controller settings are  $\{K_1, T_{i1}\} = \{1.36, 102\}$  and  $\{K_2, T_{i2}\} = \{0.24, 147\}$ .

A. Decentralized Fuzzy pre compensated PI Controller

A design philosophy reflected in Fuzzy pre compensated PI controller (FPPIC) design is that fuzzy methods can be used effectively to complement conventional control methods for performance improvement

The basic configuration of FPPIC is shown in Figure 3. It is seen that FLC is just a supplementary role to enhance the existing control system when the control conditions changes. Suppose the original PI controller is designed to reach a good performance in nominal operating case. When disturbance is acting on the system, the performance of the PI controller is no longer with in the desired range Thus the PI controller should be redesigned. However, redesigning of new PI controller to replace the existing one or tuning the parameters of PI controller are troublesome and sometimes impossible.

In this case, FLC will provide a modified control action to the existing PI/PID control system. Since FLC are easy to realize and the behavior can be redesigned by modifying the fuzzy logic rules one does not have to redesign the existing control system hardware in order to acquire satisfactory response when disturbances and noise appear. The FPPIC has obviously reduced the overshoot, undershoot, and steady state error.

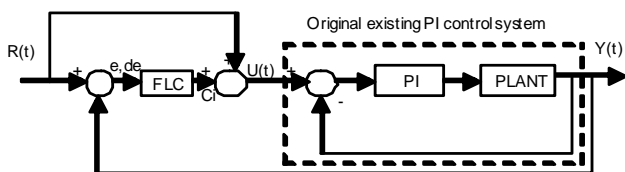


Figure 3 . The Basic Block Diagram of Fuzzy pre compensated PI control System

The decentralized fuzzy precompensated PI control structure includes two fuzzy SISO controllers. In the proposed control method for the Quadruple Tank System, two fuzzy logic controllers used separately for controlling the level outputs. The structure is shown in Figure 4.

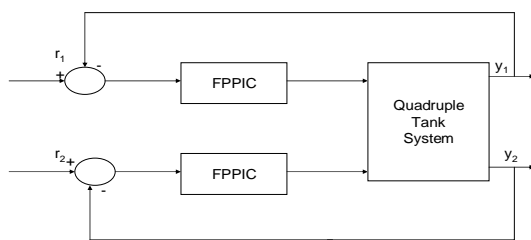


Figure 4. Decentralized control structure for minimum-phase system with two Fuzzy controllers

Basic configuration of FLC comprises of three principal components: fuzzification interface, decision making logic and a defuzzification interface.

Fuzzification

Fuzzy logic uses linguistic variables instead of numerical variables. The process of converting a numerical variable in to a linguistic variable is called fuzzification. In the present work the error and change in error of level outputs ( $h_1$  and  $h_2$ ) are taken as inputs and the pump voltages ( $v_1, v_2$ ) are the controller outputs. The error and change in error is converted into seven linguistic values namely NB, NM, NS, ZR, PS, PM and PB. Similarly controller output is converted into seven linguistic values namely NB, NM, NS, ZR, PS, PM and PB. Triangular membership function is selected and the elements of each of the term sets are mapped on to the domain of corresponding linguistic variables. The membership functions for error change in error and controller output is displayed in Figure 5 and 6.

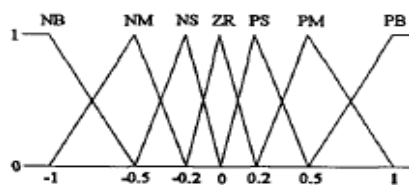


Figure 5. Membership functions of error and change in error

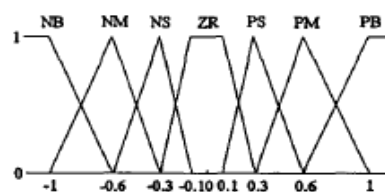


Figure 6. Membership functions of controller output

Decision Logic stage

Basically, the decision logic stage is similar to a rule base consisting of fuzzy control rules to decide how FLC works. This stage is constructed by expert knowledge and experiences. The rules are generated heuristically from the response of the conventional controller. 49 rules are derived for each fuzzy controller from careful analysis of trend obtained from the simulation of conventional controller and known process knowledge. The rules are enumerated in Table 3. The decision stage processes the input data and computes the controller outputs.

Defuzzification

The output of the rule base is converted into crisp value, this task is done by defuzzification module. Centroid method of defuzzification is considered for this

de e ci	NB	NM	NS	ZR	PS	PM	PB
NB	NB	NB	NB	NM	NS	NS	ZR
NM	NB	NB	NM	NS	NS	ZR	PM
NS	NB	NM	NS	NS	ZR	PS	PM
ZR	NM	NM	NS	ZR	PS	PM	PB
PS	NM	NS	ZR	PS	PS	PM	PB
PM	NS	ZR	PS	PS	PM	PB	PB
PB	ZR	PS	PS	PM	PB	PB	PB

application.

TABLE .3

RULE TABLE OF FUZZY LOGIC COMPENSATOR FOR LOOP 1

IV. RESULTS AND DISCUSSIONS

This section discusses the simulation studies carried out on the quadruple-tank system. To validate the performance of the designed controllers, closed-loop simulations were conducted. For the purpose of simulation, the ‘real’ process is simulated by the nonlinear state space model. Separate simulations were performed for both minimum-phase and non-minimum phase systems. The performances were compared in terms of integral error values.

A. Decentralized PI Controller

The decentralized PI control scheme gives good performance, since the chosen manipulated variable – process variable pairs have a larger relative gain. The closed loop servo responses for tank-1 and tank-2 for the minimum phase operation are shown in the Figures 7 and 8 respectively. The closed loop servo responses for tank-1 and tank-2 for the non-minimum phase operation are displayed in the Figures 9 and 10 respectively.

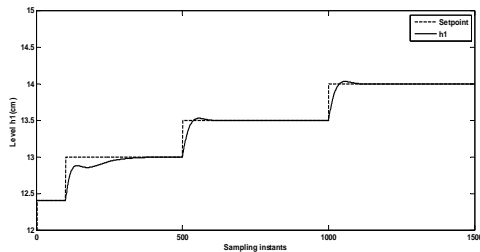


Figure 7. Servo Response of Minimum Phase System with Decentralized PI Controller for Tank 1

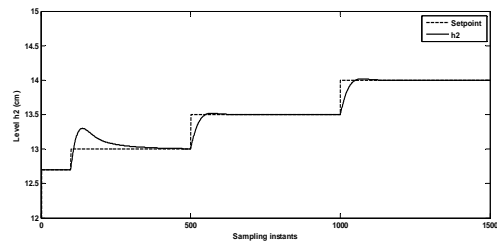


Figure 8. Servo Response of Minimum Phase System with Decentralized PI Controller for Tank 2

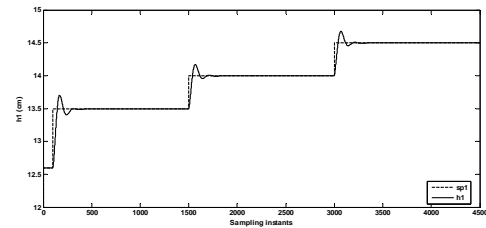


Figure 9. Servo Response of Non- Minimum Phase System with Decentralized PI Controller for Tank 1

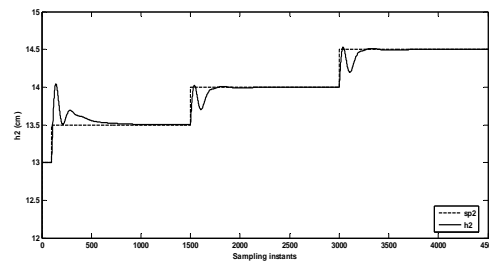


Figure 10. Servo Response of Non- Minimum Phase System with Decentralized PI Controller for Tank 2

The closed loop response of Quadruple tank System with Decentralized PI controller has oscillations and overshoot and long settling time particularly in non minimum phase operating condition.

B. Decentralized Fuzzy precompensated PI Controller

The Quadruple Tank System was simulated with Decentralized FPPIC and the closed loop servo responses for tank-1 and tank-2 with for the minimum phase operation are shown in the Figures 11 and 12 respectively. The closed loop servo responses with Decentralized FPPIC for tank-1 and tank-2 with non-minimum phase operation are displayed in the Figures 13 and 14 respectively.

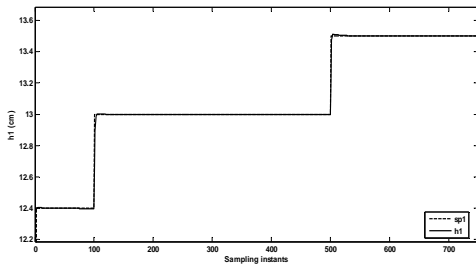


Figure 11. Servo Response of Minimum Phase System with Decentralized Fuzzy pre compensated PI controller for Tank 1

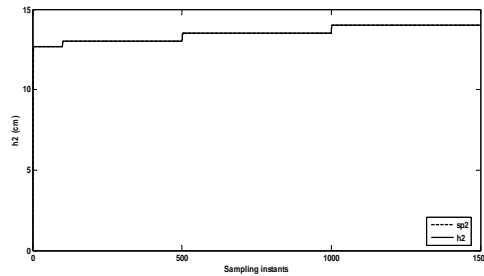


Figure 12. Servo Response of Minimum Phase System with Decentralized Fuzzy pre compensated PI controller for Tank 2

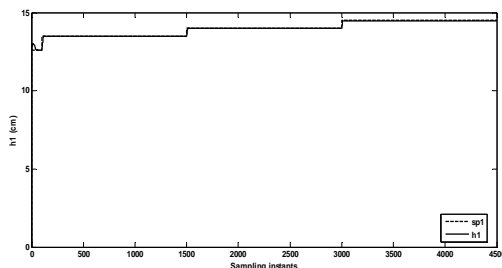


Figure 13. Servo Response of Non-Minimum Phase System with Decentralized Fuzzy pre compensated PI controller for Tank 1

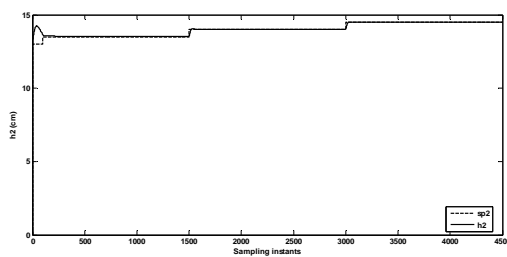


Figure 14. Servo Response of Non-Minimum Phase System with Decentralized Fuzzy pre compensated PI controller for Tank 2

The closed loop response of Quadruple tank System with Decentralized Fuzzy Pre compensated PI controller has less oscillations and overshoot. The performance of the two controllers is evaluated using performance indices namely Integral Square error (ISE) and Integral Absolute Error (IAE). A control system is considered optimal when it minimizes the above integrals. Table 4 summarizes the integral error values for the two control schemes. Decentralized Fuzzy pre

compensated PI controller has the least ISE, and IAE values.

TABLE 4.

QUANTITATIVE COMPARISON OF PERFORMANCE INDICES

Controller	Performance indices	Minimum Phase		Non-minimum phase	
		Loop 1	Loop 2	Loop1	Loop2
Decentralized PI controller	ISE	8.1	5.9	33.4	81.8
Decentralized FPPI controller	ISE	3.6	1.9	12.3	45.2
Decentralized PI controller	IAE	5.6	1.3	12.7	18.4
Decentralized FPPI controller	IAE	2.5	0.9	10.9	11.3

V. CONCLUSION

This work clearly shows the potential advantages of using Decentralized fuzzy pre compensated PI controller for a Quadruple Tank System. The control algorithm has a good set point tracking without any offset with reasonable settling time. The comparison of the two controllers reveals that decentralized fuzzy pre compensated PI controller is superior resulting in smoother controller output without oscillations which would increase the actuator life.

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