Fractal Antennas: A Novel Miniaturization Technique for Wireless Communications

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Abstract - **Wireless applications, particularly with multiple resonances, put new demands on antennas pertaining to size, gain, efficiency, bandwidth, and more. One promising approach in this regard is to use fractal geometries to find the best distribution of currents within a volume to meet a particular design goal. In the miniaturization of wire antennas it has been found that the electromagnetic coupling between wire angles limits the reduction of the resonant frequency with increasing wire length. Recent efforts by several researchers around the world to combine fractal geometry with electromagnetic theory have led to an emergence of new and innovative antenna designs. Unique properties of fractals have been exploited to develop a new class of antenna-element designs that are multi-band and compact in size and have been shown to possess several highly desirable properties, including multi-band performance, low sidelobe levels, and its ability to develop rapid beamforming algorithms based on the recursive nature of fractals. The purpose of this paper is to introduce the concept of the fractals and to provide a study and implementation of rapidly growing field of fractal antenna engineering including recent developments. The simulated results for Koch fractal antenna using AWAS and NEC are also provided.**

Index Terms – Fractals, Fractal antenna , Fractal electrodynamics

I. INTRODUCTION

In the study of antennas, fractal antenna theory is a relatively new area. The emergence of antennas with fractal geometries has given an answer to two of the main limitations stated by Werner (1999) of the classical antennas, which are the single band performance and the dependence between size and operating frequency. The term “fractal”, means broken or irregular fragments. It was originally coined by Mandelbrot (1983) to describe a family of complex shapes that possess an inherent self-similarity or self-affinity in their geometrical structure. Jaggard (1990) defined fractal electrodynamics as an area in which fractal geometry was combined with electromagnetic theory for the purpose of investigating a new class of radiation, propagation, and scattering problems. One of the most promising areas of fractal electrodynamics research is in its application to antenna theory and design. There are a variety of approaches that have been developed over the years, which can be utilized to achieve one or more of these design objectives. The development of fractal geometry came largely from an in depth study of the patterns of nature. With the advance of wireless communication systems and their increasing importance, wideband and low profile antennas are in great demand for both commercial and military applications. Multiband and wideband antennas are desirable in personal communication systems, small satellite communication terminals, and other wireless applications. Some of these applications also require an antenna to be embedded into the airframe structure. Traditionally, a wideband antenna in the low frequency wireless bands can only be achieved with heavily loaded wire antennas, which usually means that different antennas are needed for different frequency bands. Recent progress in the study of fractal antennas suggests some attractive solutions for using a single small antenna operating in several frequency bands. The self-similar properties of certain fractals result in a multiband behavior of the antennas while, the highly convoluted shape of these fractals makes possible the reduction in size, and consequently in mass and volume, of certain antennas as investigated by Puente *et al.* (1998). These reductions can make possible to combine multimedia, communication and tele-detection functionalities in a reduced space like a handy phone, a wristwatch or a credit card e.g. a fractal antenna can provide GPS (Global Positioning System) services within a conventional mobile cellular phone. Since Hertz times, the design of electrically small antennas has always been a topic of great interest, related first to the development of radiotelegraphy and radio broadcasting. In the last few years, the fast growing development of mobile communication brought the need for devices that require their components to be ever smaller and lighter, capable of adjusting its frequency of operation and to operate in a multiband mode. Some recent results by Puente (1998) and Puente (2001) showed that fractal antennas have excellent multiband properties and low resonant frequencies. An overview of the early work on these antennas is summarized by Werner (1999). Radiation efficiency and impedance bandwidth decrease with the size of the antenna, making small antennas inefficient by nature, for these effects are accompanied by high currents in the conductors, high ohmic losses and large values of energy stored in the antenna near field. The inefficient performance of small antennas is summarized by the high values of its quality factor *Q*, as predicted by the fundamental limit and stated by Chu (1948) and McLean (1996). This limit was set assuming that an infinitesimally small antenna radiates only a TE_{10}or TM_{01}spherical mode that depends on the electric size of the antenna ka, k is the wave number at resonance and a the radius of the smallest sphere that encloses the antenna. Real antennas radiate more reactive modes, contributing to larger *Q* values. Lowering the Q factor of an electrically small antenna, defined as $ka < 1$, is only possible by a proper use of the volume that surrounds it with the objective of exciting only a TE_{10}or TM_{01} mode. Hence in order to meet the following attributes for antenna designs, i.e. the compact size, low profile, conformal and multiband or broadband, a number of approaches for designing multi-band antennas have been summarized by Maci (1997). Recently, the possibility of developing antenna designs that exploit in some way the properties of fractals to achieve these goals, at least in part, has attracted a lot of attention. Traditional approaches to the analysis and design of antenna systems have their foundation in Euclidean geometry. There has been a considerable amount of recent interest with the possibility of developing new types of antennas that employ fractal rather than Euclidean geometric concepts in their design. This new and rapidly growing field of research has been referred as fractal antenna engineering. Because fractal geometry is an extension of classical geometry, its recent introduction provides engineers with the unprecedented opportunity to explore a virtually limitless number of previously unavailable configurations for possible use in the development of new and innovative antenna designs. There are primarily two active areas of research in fractal antenna engineering. These include the study of fractal-shaped antenna elements and the use of fractals in the design of antenna arrays. The purpose of this article is to provide an overview of recent developments in the theory and design of fractal antenna elements, as well as fractal antenna arrays. Fractals are space-filling contours, meaning electrically large features can be efficiently packed into small areas. Since the electrical lengths play an important role in antenna design, this efficient packing can be used as a viable miniaturization technique. Since these antennas are becoming increasingly popular and for that reason, it is a point of careful study of electrical performance versus technological complexity trade-offs to provide answers about the potential interest of fractal antennas.

II. FRACTAL CONCEPTS

‘Fractal’ was first defined by Benoit Mandelbrot (1983) as a way of classifying structures whose dimensions were not whole numbers.

These geometries have been used previously to characterize unique occurrences in nature that were difficult to define with Euclidean geometries, including the length of coastlines, the density of clouds, and branching of trees. Most fractal objects have self-similar shapes although there some fractal objects exist that are hardly self-similar at all and also have infinite complexity and detail, that is, the complexity and detail of the fractals remain no matter how far you "zoom-in," as long as you are zooming in on the right location. Fractals can model nature very well. Fractals can be divided into many types, Fig. 1 show some examples. Many theories and innovative applications for fractals are being developed. Fractals have been applied in image compression, in the creation of music from pink noise, and in the analysis of high altitude lightning phenomena.

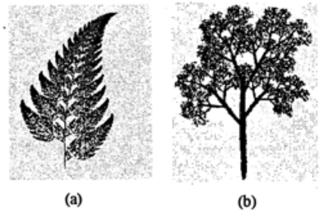


Figure1. Examples of fractals modeling nature (a)tree (b) plant

III. THE GEOMETRY OF FRACTALS

The geometry of fractals is important because the effective length of the fractal antennas can be increased while keeping at total area same. The shape of the fractal antenna can be formed by an iterative mathematical process, called as Iterative function systems (IFS). IFS represent an extremely versatile method for conveniently generating a wide variety of useful fractal structures summarized by Bamsley (1993). These iterated function systems are based on the application of a series of affine transformations, w, defined by: $w\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}e\\f\end{pmatrix}$or $w(x,y)=(ax+by+e, cx+dy+f)$ where *a*,*b*,*c*,*d*,*e* and *f* are real numbers. Hence the affine transformation, w , is represented by six parameters with *a*, *b*, *c*, *d*, *e* as control rotation and scaling while *e* and *f* as control linear translation. Now suppose we consider $w_1, w_2, \ldots\ldots, w_N$as a set of affine linear transformations, A be the initial geometry, then a new geometry, produced by applying the set of transformations to the original geometry, *A*, and collecting the results from $w_1(A), w_2(A), \ldots\ldots w_N(A)$, this can be represented by $W(A)=\bigcup_{n=1}^{N} w_n(A)$ where W is known as the Hutchinson operator. A fractal geometry can be obtained by repeatedly applying W to the previous geometry. For example, if the set A_0 represents the initial geometry, then $A_1=W(A_0), A_2=W(A_1), \ldots\ldots A_{k+1}=W(A_k)$. An iterated function system generates a sequence that converges to a final image, A_∞ , in such a way that $W(A_\infty)= A_\infty$. This image is called the attracter of the iterated function system and represents a fixed point of W, Fig. 2 shows a Koch curve, Fig. 3 shows a Sierpinski fractal and Fig. 4 shows Koch Island. It should be noted that applying several of these transformations in a recursive way, the self-similar fractal is obtained. Infact, self-similarity can also be understood as the property by which the fractal is found inside the fractal itself but at smaller scale. Fractal structures can be analyzed using the integral equation methods (IE), in conjunction with the well-known methods of moments (MOM), which splits the fractal geometry in the basis functions. Iterated function systems have proven to be a very powerful design tool for fractal antenna engineers. This is primarily because they provide a general framework for the description, classification, and manipulation of fractals according to the studies by Bamsley (1993).

IV. FRACTALS AS SPACE FILLING GEOMETRIES

A fractal is mathematically defined to be infinite in intricacy, this is not desirable if antennas are to be fabricated using these geometries. For example, the complexity and repetition of a cloud does not extend to infinitely small or large scales, but can be approximated as doing so for a certain band of scales. From the scale of human perception, a cloud does seem to be infinitely complex in larger and smaller scales. The resulting geometry after truncating the complexity is called a "prefractal". A prefractal drop the intricacies that are not distinguishable in the particular applications. While Euclidean geometries are limited to points, lines, sheets, and volumes, fractals include the geometries that fall between these distinctions, a fractal can be a line that approaches a sheet. The line can meander in such a way as to effectively almost fill the entire sheet. These space-filling properties lead to curves that are electrically very long, but fit into a compact physical space and can lead to the miniaturization of antenna elements. As mentioned earlier and indicated by Gianviffwb (2002) that prefractals drop the complexity in the geometry of a fractal that is not distinguishable for a particular application. For antennas, this can mean that the intricacies that are much, much smaller than a wavelength in the band of useable frequencies can be dropped out. This now makes this infinitely complex structure, which could only be analyzed mathematically, manufacturable.

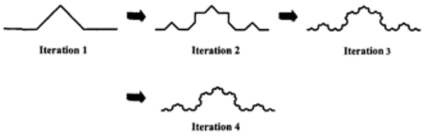


Figure 2. A four iteration Koch fractal

Figure 3. A three iteration Sirepinski fractal

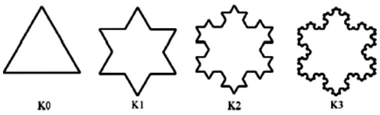


Figure 4. A three iteration Koch island

V. MINITURIZATION OF ANTENNAS

Wire antennas miniaturization is usually based in packing a long wire inside a small volume with the aim to achieve the smallest antenna having a given resonant frequency or, equivalently, achieving the lowest resonant frequency of an antenna having a fixed size. In principle, it is expected that the longer the wire length, the lower the resonant frequency. This can be explained by the fact that the degree of coupling between parallel wire segments with opposite current vectors causes a significant reduction in the effective length of the total wire, and therefore an increase in the resonant frequency. The effect can be explained with the help of Koch fractal curve is taken to understand the behavior of the resonant frequency of fractal antennas as a function of the antenna geometry and wire length. As observed by Puente (1998) with an increase of the wire length of a Koch fractal there is a in the resonant frequency which results due to the coupling between sharp angles at curve segment junctions. These angles radiate a spherical wave with phase center at the vertex as shown in Fig. 5.

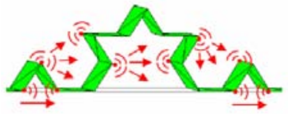


Figure 5. Signal coupling at angles in koch curve fractal

Each angle not only radiates, but also receives the signal radiated by other angles. As a consequence, part of the signal does not follow the wire path, but takes "shortcuts" that start at a radiating angle. The length of the path traveled by the signal is, therefore, shorter than the total wire length.

VI. FRACTAL ANTENNA ELEMENTS

As already mentioned, the general concepts of fractals can be applied to develop various antenna elements and thus allows for smaller, resonant antennas that are multiband and may be optimized for gain. When antenna elements or arrays are designed with the concept of self-similarity for most fractals, they can achieve multiple frequency bands because different parts of the antenna are similar to each other at different scales. Application of the fractional dimension of fractal structure leads to the gain optimization of wire antenna and the self-similarity makes it possible to design antennas with very wideband performance. The first application of fractals to antenna design was thinned fractal linear and planar arrays were studied by Kim (1986) and Wemer (1996). They obtained wideband arrays and Multiband performance by arranging elements in a fractal pattern to reduce number of elements in an array. Cohen (1997) was the first to develop an antenna element using the concept of fractals. He demonstrated that the concept of fractal could be used to significantly reduce the antenna size without degenerating the performance. Puente *et al.* (1998) demonstrated the multiband capability of fractals by studying the behavior of the Sierpinski monopole and dipole. The Sierpinski monopole displayed a similar behavior at several bands for both the input return loss and radiation pattern. Jaggard (1990) had also explored other fractals to obtain multi-band or ultra-wideband antennas. In other designs, fractal structures are used to achieve a single very wideband response, such as the printed circuit fractal loop.

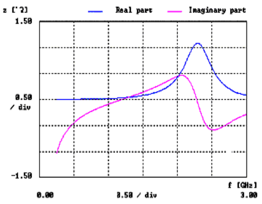
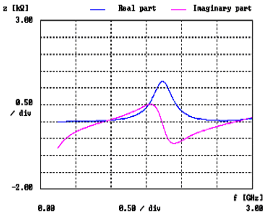
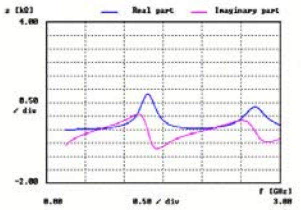
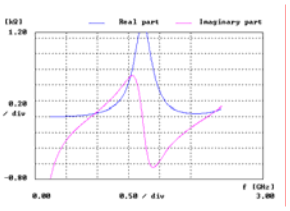
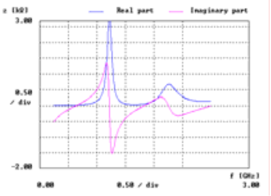
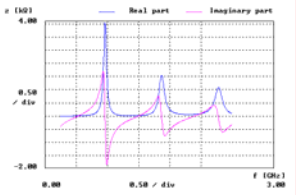
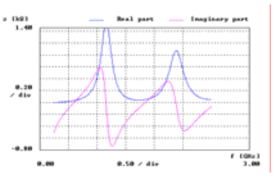
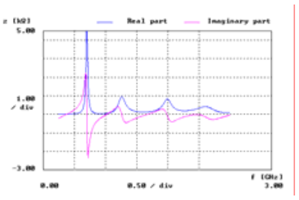
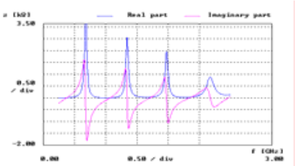
A. *Koch monopole and dipole*

The application of fractal geometry to the design of wire antenna elements was first reported in a series of articles by Cohen (1995) and Cohen (1996). These articles introduce the notion of fractalizing the geometry of a standard dipole or loop antenna which can be accomplished by systematically bending the wire in a fractal way, so that the overall arc length remains the same, and the size is correspondingly reduced with the addition of each successive iteration. The Koch curve has been used to construct a monopole and a dipole in order to reduce antenna size shown in Fig. 2. It is generated by replacing the middle third of each straight section with a bent section of wire that spans the original third. It has been observed that the effective length of a Koch fractal antenna increases by $(4/3)^n$ for each iteration, where n is the number of iterations. The miniaturization of the antennas shows a greater degree of effectiveness for the first several iterations. Properties of the Koch fractal monopole were later considered in Puente (1998) and it was shown that the electrical performance of Koch fractal monopoles is superior to that of conventional straight wire monopoles, especially when operated in the small-antenna frequency regime. In this paper, the results for Koch monopole of length 6cm are found to be in accordance with Puente (2001). The corresponding results for different lengths 8cm and 12cm with first two iterations are listed in Table 1 and Table 2. The feed point for all of the antennas was located at the ground plane and wire was considered to be perfect conductor with a radius of 100µm is considered. Through the plots it can be observed that as the number of iterations of the fractal iterations increases, the resonant frequency decreases.

TABLE I Effective length of Koch monopole

Antenna	Effective length (6cm)	Effective length (8cm)	Effective length (12cm)
K0	6	8	12
K1	8	10.67	16
K2	10.67	14.22	21.33
K3	14.22	18.96	28.44

TABLE II: Input impedances for the Koch monopole

S.no.	Length of Koch monopole (cm)	K0	K1	K2
1.	6			
2.	8			
3	12			

B. *Koch loop and Minkowski loop*

Loop antennas, Fig. 6 can be well understood using a variety of Euclidean geometries. Resonant loop antennas require a large amount of space and small loops have very low input resistance a fractal island can be used as a loop antenna to overcome these drawbacks. Minkowski loops were originally investigated by Cohen (1995) and Cohen (1996). Both types of fractal loops have the same characteristic that the perimeter increases to infinity while maintaining the volume occupied. This increase in length decreases the required volume occupied for the antenna at resonance, for a small loop this increase in length improves the input resistance. By raising the input resistance, the antenna can be more easily matched to a feeding transmission line.

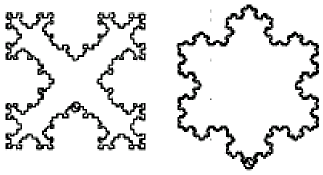

(a) (b)

Figure 6. (a) Koch Loop (b) Minkowski loop

C. Sierpinski monopole and dipole

The Sierpinski gasket is named after the Polish mathematician Sierpinski who described some of the main properties of this fractal shape in 1916. The original gasket is constructed by subtracting a central inverted triangle from a main triangle shape, Fig. 7. The self-similarity properties of the fractal shape are translated into its electromagnetic behavior and results in a multiband antenna. A multiband fractal monopole antenna, based on the Sierpinski gasket, was first introduced by Puente *et al.* (1996) with a flare angle of α and a self-similarity scale factor of δ .

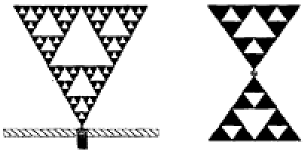


Figure7.Sierpinski monopole and dipole

A scheme for modifying the spacing between the hands of the Sierpinski monopole was subsequently presented by Puente *et al.* in (1996). It was demonstrated by Hohlfeld (1999) that the positions of the multiple bands may be controlled by proper adjustment of the scale factor used to generate the Sierpinski antenna. It was found that a variation in the flare angle of the antenna translated into a shift of the operating bands, as well as into a change in the input impedance and radiation patterns. Fast iterative network models that are useful for predicting the performance of Sierpinski fractal antennas were developed by Borja (1998) and Puente (2000).

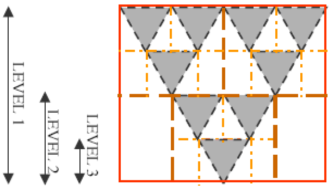


Figure 8. Sierpinski gasket with different parameters

D. Fractal Tree

Various fractal tree structures have been explored as antenna elements and can achieve multiple wideband performance and reduce antenna size, Fig. 9 show a stage 4 ternary fractal tree. The multiband characteristics of a deterministic fractal tree structure were considered by Xu (1999). It was found that these fractal tree antennas have a multiband behavior with a denser hand distribution than the Sierpinski antenna. The multiband and wideband properties of printed fractal branched antennas were studied by Sindoy (1999).

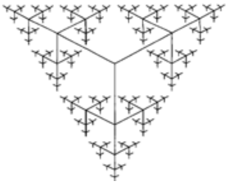


Figure 9. A stage 4 Fractal tree

The space-filling properties of two-dimensional and three-dimensional fractal trees were suggested by Gianvittorio (2000) as good candidates for application to the design of miniaturized antennas in which it was shown that a reduction in the resonant frequency of a standard dipole can be achieved by end-loading it with two-dimensional or three-dimensional tree-like fractal structures. This decrease in resonant frequency was shown to asymptotically approach a limit as the numbers of iterations increased. Some of ways to improve antenna miniaturization techniques were discussed by Sindou (1999), employing fractal tree geometries as end-loads by increasing the density of branches (i.e., by using trees with a higher fractal dimension).

E. Fractal patch antennas

Borja (2000) proposed a design methodology for a multiband Sierpinski microstrip patch antenna. A technique was introduced to improve the multiband behavior of the radiation patterns by suppressing the effects of high order modes. High-directivity modes in a Koch-island fractal patch antenna were studied by Romeu (2001). It was shown that a patch antenna with a Koch fractal boundary exhibits localized modes at a certain frequency above the fundamental mode, which can lead to broadside directive patterns. Also, the localized modes were also observed in a waveguide having Koch fractal boundaries. Some additional applications of fractal concepts to the design of microstrip-patch antennas were considered and studied by Yeo (2001) and Gianvittorio (2001). The Cantor slot patch is another example of multiband fractal structure. This type of patch has been applied in multiband microstrip antennas.

F. Printed circuit fractal antennas

Printed circuit antennas are desired in many instances due to the space constrains in the modem electronic devices. The printed circuit antennas are useful for the easiness of construction and for the reduced occupied space. The printed circuit fractal loop antenna is designed to achieve ultra wideband or multiple wideband performance and to reduce the antenna dimensions. The antenna has a constant phase center, can be manufactured using printed circuit techniques, and is readily conformable to an airframe or other structure.

VII. FRACTAL ANTENNA ARRAYS

The concept of the fractal can be applied in design and analysis of arrays by either analyzing the array using fractal theory, or placing elements in fractal arrangement, or both. Fractal arrangement of array elements can produce a thinned array and achieve multiband performance. Properties of random fractals were used by Jaggard (1986) to develop a design methodology for quasi-random arrays. Random fractals were also used to generate array configurations that were somewhere between completely ordered (i.e., periodic) and completely disordered (i.e., random). The main advantage of this technique is that it yields sparse arrays that possess relatively low sidelobes (a feature associated with periodic arrays, but not random arrays), and which are also robust (a feature associated with random arrays, but not periodic arrays). A family of nonuniform arrays, known as Weierstrass arrays have the property that their element spacings and current distributions are self-scalable and can be generated in a recursive fashion. Synthesis techniques for fractal radiation patterns were developed by Werner (1996), based on the self-scalability property.

A. Multi-Band Fractal Arrays

A design methodology for multi-band Weierstrass fractal arrays was introduced by Wemer (1997). The application of fractal concepts to the design of multiband Koch arrays, as well as multiband and low-sidelobe Cantor arrays were discussed by Puente (1996). A simplified Koch multiband array, using windowing and quantization techniques, was presented by El-Khamy (2000). It was recently shown by Werner (2000) that the Weierstrass-type and the Koch type of multiband arrays are actually special cases of a more general unified family of self-scalable multi-band arrays.

B. Cantor, Sierpinski Carpet, and Related Arrays

The radiation characteristics of planar concentric ring Cantor arrays were investigated by Wemer (1997). Cantor linear array is based on a Cantor set with a number of design variables. When thinned, these arrays have a performance that is superior to their periodic counterparts and appear similar to or better than their random counterparts for a moderate number of elements. Planar fractal array configurations, based on Sierpinski carpets, were also studied by Baldacci (2001). The fact that Sierpinski carpet and related arrays can be generated recursively (i.e., via successive stages of growth starting from a simple generating array) has been exploited in order to develop rapid algorithms for use in efficient radiation-pattern computations and adaptive beamforming, especially for arrays with multiple stages of growth that contain a relatively large number of elements, Fig. 10 shows the first four stages of a linear Cantor array with dark gray dipoles represent physical elements while the light gray representing the virtual elements.

Figure 10. A four stage linear Cantor array

VIII. CONCLUSIONS

Through characterizing the fractal geometries and the performance of the antennas, it can be summarized that increasing the fractal dimension of the antenna leads to a higher degree of miniaturization. Also it is possible to use fractal structure to design small size, low profile, and low weight antennas. Applications of fractal geometry are becoming increasingly widespread in the fields of science and engineering. This paper presented a comprehensive overview of fractal antenna engineering. The analysis of a Koch monopole is also done and the results showed that as the number of iterations is increased, there is an increase in the effective length and decrease in resonant frequency. In the future, one area of development in fractal antenna is to implement fractal antennas into current technologies in practical situations, another area of interest worth pursuing is to analyze the mathematical aspects of fractals to correlate their improved characteristics as antennas with their unique geometrical properties. The field of fractal antenna engineering is still in the relatively early stages of development, with the anticipation of much more innovative advancement to come over the months and years ahead.

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