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# Reactive Power Planning using Differential Evolution: Comparison with Real GA and Evolutionary Programming

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***Abstract*— This paper proposes an application of Differential Evolution (DE) to Reactive Power Planning (RPP) problem. Simulation results, compared with those obtained by using the Real coded Genetic Algorithm (RGA) and Evolutionary Programming (EP) are presented to show that the present method is better for power system planning. In the case of optimization of non-continuous and non-smooth function, DE gives much better results than RGA and EP. The proposed approach has been used in the IEEE 30-bus system. The comprehensive simulation results show a great potential for applications of DE in power system economical and secure operation, planning and reliability assessment.**

***Index Terms*—Power systems, Differential Evolution, Real coded Genetic Algorithm, Evolutionary Programming, Reactive Power Planning.**

## I. Introduction

The reactive power planning (RPP) is one of the most complex problems of power systems as it requires the simultaneous minimization of two objective functions. The first objective deals with the minimization of operation cost by reducing real power loss and improving the voltage profile. The second objective minimizes the allocation cost of additional reactive power sources. RPP is a nonlinear optimization problem for a large scale system with a lot of uncertainties. During the last decade there has been a growing concern in the RPP problems for the security and economy of power systems [1-12]. Conventional calculus-based optimization algorithms have been used in RPP for years [l-4]. Conventional optimization methods are based on successive linearization and use the first and second differentiations of objective function and its constraint equations as the search directions. The conventional optimization methods are good enough for the optimization problems of deterministic quadratic objective function which has only one minimum. However, because the formulae of RPP problem are hyper quadric functions, such linear and quadratic treatments induce lots of local minima. The conventional optimization methods can only lead to a local minimum and sometimes result in divergence in solving RPP problems.

Over the last decade, Evolutionary Algorithms (EAs), such as, Genetic Algorithms (GA), Evolutionary Programming (EP) and Differential Evolution(DE) have been extensively used to solve the non-linear problems. [5-12]. The Evolutionary Algorithms have the following advantages compared to conventional methods.

1. It uses population of potential solutions, not single point, which can move over hills and across valleys to discover a globally optimal point.
2. Because the computation for each individual in the population is independent of others, it has inherent parallel computation ability.
3. It uses payoff (fitness or objective functions) information directly for the search direction, not derivatives or other auxiliary knowledge, therefore can deal with non-smooth, non- continuous and non-differentiable functions that are the real-life optimization problems. RPP is one of such problems.
4. It uses probabilistic transition rules to select generations, not deterministic rules, so it can search a complicated and uncertain area to find the global optimum.

This paper proposes an application of Differential Evolution to solve the RPP problem. DE is a mathematical global optimization method for solving multidimensional functions. Main idea of DE is to generate trial parameter vectors using vector differences for perturbing the vector population.

The proposed method is compared with RGA and EP. In RGA, crossover and mutation operators are applied directly to real parameter values and decision variables can be directly used to compute the fitness values. RGA is based on the mechanics of natural selections-selection, crossover and mutation, whereas EP is based on mutation, competition and evolution.

The proposed approach has been used in the IEEE 30-bus system [3] which consists of six generator buses, 21 load buses and 41 branches of which four branches, (6,9), (6,10), (4,12) and (28,27) are under load tap-setting transformer branches. The reactive power source installation buses are buses 7, 12, 21 and 30. There are totally 14 control variables.

## II. PROBLEM FORMULATION

List of Symbols

$N_l$ = set of numbers of load level durations
$N_E$ = set of branch numbers
$N_c$ = set of numbers of possible VAR source installment buses
$N_i$ = set of numbers of buses adjacent to bus *i*, including bus *i*
$N_{PQ}$ = set of PQ - bus numbers
$N_g$ = set of generator bus numbers
$N_T$ = set of numbers of tap - setting transformer branches
$N_B$ = set of numbers of total buses
$h$ = per - unit energy cost
$d_l$ = duration of load level *1*
$g_k$ = conductance of branch *k*
$V_i$ = voltage magnitude at bus *i*
$\theta_{ij}$ = voltage angle difference between bus *i* and bus j
$e_i$ = fixed VAR source installment cost at bus *i*
$C_{ci}$ = per - unit VAR source purchase cost at bus *i*
$Q_{ci}$ = VAR source installed at bus *i*
$Q_i$ = reactive power injected into network at bus *i*
$G_{ij}, B_{ij}$ = mutual conductance and susceptance between bus *i* and bus *j*
$G_{ii}, B_{ii}$ = self conductance and susceptance of bus *i*
$Q_{gi}$ = reactive power generation at bus *i*
$T_k$ = tap -setting of transformer branch *k*
$N_{Vlim}$ = set of numbers of buses of voltage overlimits
$N_{Qglim}$ = set of numbers of buses of reactive power generation overlimits

The objective function in RPP problem comprises two terms [14]. The first term represents the total cost of energy loss as follows:

$$W_C = h\sum_{l\in N_l} d_l P_{loss,l} \qquad (1)$$

where $P_{loss,l}$ is the network real power loss during the period of load level *1*. The $P_{loss,l}$ can be expressed in the following equation in the duration $d_l$ :

$$P_{loss} = \sum_{\substack{k\in N_E \\ k\in(i,j)}} g_k\left(V_i^2 + V_j^2 - 2V_iV_j Cos\,\theta_{ij}\right) \qquad (2)$$

The second term represents the cost of VAR source installments which has two components, namely, fixed installment cost and purchase cost:

$$I_C = \sum_{i\in N_C}\left(e_i + C_{ci}Q_{ci}\right) \qquad (3)$$

The objective function, therefore, can be expressed as follows:

$$\begin{aligned}
&\min f_C = I_C + W_C \\
&s.t. \\
&0 = Q_i - V_i\sum_{j\in N_i} V_j\left(G_{ij}Sin\,\theta_{ij} - B_{ij}Cos\,\theta_{ij}\right) && i\in N_{PQ} \\
&Q_{ci}^{\min} \le Q_{ci} \le Q_{ci}^{\max} && i\in N_c \\
&Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max} && i\in N_g \\
&V_i^{\min} \le V_i \le V_i^{\max} && i\in N_B \\
&T_k^{\min} \le T_k \le T_k^{\max} && k\in N_T
\end{aligned} \qquad (4)$$

where reactive power flow equations are used as equality constraints; VAR source installment restrictions, reactive power generation restrictions, transformer tap-setting restrictions and bus voltage restrictions are used as inequality constraints. $Q^{min}{}_{ci}$ can be less than zero and if $Q_{ci}$ is selected as a negative value, say in the light load period, variable reactance should be installed at bus i. The transformer tap setting *T*, generator bus voltages $V_g$ and VAR source installments $Q_c$ are control variables so they are self restricted. The load bus voltages $V_{load}$ and reactive power generations $Q_g$ are state variables, which are restricted by adding them as the quadratic penalty terms to the objective function to form a penalty function. Equation (4) is therefore changed to the following generalized objective function:

$$\min F_C = f_C + \sum_{i\in N_{V\lim}} \lambda_{Vi}\left(V_i - V_i^{\lim}\right)^2 + \sum_{i\in N_{Qg\lim}} \lambda_{Qgi}\left(Q_{gi} - Q_{gi}^{\lim}\right)^2$$

*s.t.*

$$0 = Q_i - V_i\sum_{j\in N_i} V_j\left(G_{ij}Sin\,\theta_{ij} - B_{ij}Cos\,\theta_{ij}\right) \qquad i\in N_{PQ}$$

where $\lambda_{vi}$ and $\lambda_{Qgi}$ are the penalty factors which can be increased in the optimization procedure; $V^{lim}{}_i$ and $Q^{lim}{}_{gi}$ are defined in the following equations:

$$V_i^{\,lim} = \begin{cases} V_i^{\,min} & if\ \ V_i < V_i^{\,min} \\ V_i^{\,max} & if\ \ V_i > V_i^{\,max} \end{cases}$$

$$Q_{gi}^{\,lim} = \begin{cases} Q_{gi}^{\,min} & if\ \ Q_{gi} < Q_{gi}^{\,min} \\ Q_{gi}^{\,max} & if\ \ Q_{gi} > Q_{gi}^{\,max} \end{cases}$$

It can be seen that the generalized objective function $F_C$ is a nonlinear and non-continuous function. Furthermore, it contains a lot of uncertainties because of uncertain loads and other factors.

## III. DIFFERENTIAL EVOLUTION(DE)

DE was introduced by Storn and Price in the year 1995[13]. It uses parameter vectors as individuals in a population. The key element distinguishing DE from other population based techniques is differential mutation operator.

Main Steps of the DE Algorithm

Initialization: First, all parameter vectors in a population are randomly initialized and evaluated using the fitness function. The initial NP D-dimensional parameter vectors is $x_{ji,G}$ where $i$=1,2,...,NP and $j$ = 1, 2, ..., D. NP is number of population vectors. D is dimension and G is generation.

Mutation: DE generates new parameter vectors by adding the weighted difference between two parameter vectors to a third vector. For each target vector $x_{i,G}$, $i$ = 1,2,…,NP, a mutant vector is generated according to:

$$\underline{v}_{i,G+1} = x_{r1,G} + F\,(x_{r2,G} - x_{r3,G})$$

where $\underline{v}_{i,G+1}$ is a mutant vector; *r1,r2,r3* are the randomly selected, mutually different vectors; F is a real and constant factor [0, 2] which controls the amplification of the differential variation

Recombination: The mutated vector's parameters are then mixed with the parameters of another predetermined vector, the target vector, to yield the so-called trial vector. Choosing a subgroup of parameters, $j$ (or a set of crossover points) for mutation is similar to a process known as crossover in genetic algorithms or evolution strategies. Crossover is introduced to increase the diversity of the perturbed parameter vectors. The trial vector:

$$\underline{u}_{i,G+1} = (\underline{u}_{1i,G+1}, \underline{u}_{2i,G+1}, \ldots, \underline{u}_{Di,G+1})$$

is formed by

$$\underline{u}_{ji,G+1} = \begin{cases} \underline{v}_{ji,G+1} & \text{if } (randb(j) \le CR) \text{ or } j = rnbr(i) \\ x_{ji,G} & \text{if } (randb(j) > CR) \text{ and } j \ne rnbr(i) \end{cases},$$

$$j = 1,2,\ldots,D.$$

where *randb(j)* is the *j*th evaluation of a uniform random number generator with outcome [0, 1], *CR* is the crossover constant [0, 1] which has to be determined by the user, *rnbr(i)* is randomly chosen index from 1..D which ensures that $\underline{u}_{i,G+1}$ gets at least one parameter from $\underline{v}_{i,G+1}$

Selection: If the trial vector yields a lower cost function value than the target vector, the trial vector replaces the target vector. In other words, the offspring replaces the parent if it is fitter. Otherwise, the parent survives and is passed on to the next iteration of the algorithm

## IV. COMPARISON METHODS

Real coded Genetic Algorithm (RGA) [15]:

Tournament Selection: In the tournament selection, tournaments are played between two solutions and the better solution is chosen and placed in the mating pool.

Simulated Binary Crossover (SBX): It works with two parent solutions and creates two offspring. The Offspring are calculated as,

$$x_i^{(1,t+1)} = 0.5\left[\left(1+\beta_{qi}\right)x_i^{(1,t)} + \left(1-\beta_{qi}\right)x_i^{(2,t)}\right]$$
$$x_i^{(2,t+1)} = 0.5\left[\left(1-\beta_{qi}\right)x_i^{(1,t)} + \left(1+\beta_{qi}\right)x_i^{(2,t)}\right]$$

Where,

$$\beta_{qi} = \begin{cases} \left(2u_i\right)^{\frac{1}{\eta_c+1}}, & if\ \ u_i \le 0.5; \\ \left(\dfrac{1}{2\left(1-u_i\right)}\right)^{\frac{1}{\eta_c+1}}, & otherwise \end{cases}$$

Polynomial Mutation: Here, the probability distribution is a polynomial function

$$y_i^{(1,t+1)} = x_i^{(1,t+1)} + \left(x_i^{(U)} - x_i^{(L)}\right)\delta_i$$

where, δi is calculated from the polynomial probability distribution

$$\delta_i = \begin{cases} \left(2r_i\right)^{\frac{1}{\eta_m+1}} - 1, & if\ \ r_i \le 0.5; \\ 1-\left[2\left(1-r_i\right)\right]^{\frac{1}{\eta_m+1}}, & if\ \ r_i \ge 0.5. \end{cases}$$

Evolutionary Programming (EP) [11, 15]:

Initialization: The initial control variable population is selected randomly. The fitness score is obtained by running P-Q decoupled power flow.

Statistics: The maximum fitness, minimum fitness, sum of fitness and average fitness of this generation are calculated.

Mutation: Each parent population is mutated and the corresponding fitness is obtained by running power flow. A combined population is formed with the old generation and the mutated old generation.

Competition: Each individual in the combined population has to compete with some other individuals to get its chance to be transcribed to the next generation.

Determination: The convergence of maximum fitness to minimum fitness is checked. If the convergence condition is not met, the mutation and competition processes will run again. If it converges, the program will check over limits of state variables. If there is no over limit, the program stops. If one or more state variables exceed their limits, the penalty factors of these variables will be increased, and then another loop of the process will start.

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## V. NUMERICAL RESULTS

In this section, IEEE 30-bus system [3] has been used to show the effectiveness of the algorithm. The network consists of 6 generator-buses, 21 load-buses and 41 branches, of which four branches, (6, 9), (6, 10), (4, 12) and (28, 27), are under load-tap setting transformer branches. The parameters and variable limits are listed in Table I.

All power and voltage quantities are per-unit values and the base power is used to compute the energy cost.

Two cases have been studied. Case 1 is of light loads whose loads are the same as [3]. Case 2 is of heavy loads whose loads are twice as those of Case 1. The duration of the load level is 8760 hours in both the cases.

### 5.1 Initial Condition

All transformer taps are set to 1.0. The initial generator voltages in Case 1 are set to 1.0. However in Case 2, because of heavy loads, all reactive power generations are out of their maximum limits. The generator bus voltages are set in different values to share the loads evenly among the generators as in Table II. The loads are given as,

*Case 1:* $P_{load}$ = 2.834 and $Q_{load}$ = 1.262
*Case 2:* $P_{load}$ = 5.668 and $Q_{load}$ = 2.524

The initial generations and power losses are obtained as in Table III.

### 5.2 Optimal Results and comparison

The optimal generator bus voltages, transformer tap-settings, VAR source installments, generations and power losses are obtained as in Tables IV-VII.

TABLE I
PARAMETERS AND LIMITS

| $S_B$ (MVA) | H ($/kWh) | $e_i$ ($) | $C_{ci}$ ($/kVAR) |
|---|---|---|---|
| 100 | 0.06 | 1000 | 30 |

| Reactive Power Generation Limits | | | | |
|---|---|---|---|---|
| Bus | 2 | 5 | 8 | 13 |
| $Q^{max}_g$ | 0.5 | 0.4 | 0.4 | 0.155 |
| $Q^{min}_g$ | -0.4 | -0.4 | -0.1 | -0.078 |

| Voltage and Tap-setting Limits | | | | | |
|---|---|---|---|---|---|
| $V^{max}_g$ | $V^{min}_g$ | $V^{max}_{load}$ | $V^{min}_{load}$ | $T^{max}_k$ | $T^{min}_k$ |
| 1.1 | 0.9 | 1.05 | 0.95 | 1.05 | 0.95 |

| Var source Installments and Voltage limits | | | |
|---|---|---|---|
| $Q^{max}_c$ | $Q^{min}_c$ | $V^{max}_c$ | $V^{min}_c$ |
| 0.36 | -0.12 | 1.05 | 0.95 |

TABLE II
GENERATOR BUS VOLTAGE SETTINGS IN CASE 2

| Bus | 1 | 2 | 5 | 8 | 11 | 13 |
|---|---|---|---|---|---|---|
| V | 1.09 | 1.05 | 1.0 | 0.965 | 1.0 | 1.0 |

TABLE III
GENERATIONS AND POWER LOSSES

| | $P_g$ | $Q_g$ | $P_{loss}$ | $Q_{loss}$ |
|---|---|---|---|---|
| Case 1 | 2.88939 | 1.11141 | 0.05544 | 0.08253 |
| Case 2 | 5.92719 | 3.31984 | 0.259171 | 1.02973 |

TABLE IV
GENERATOR BUS VOLTAGES

| Bus | | 1 | 2 | 5 | 8 | 11 | 13 |
|---|---|---|---|---|---|---|---|
| Case 1 | EP [11] | 1.074 | 1.065 | 1.043 | 1.042 | 1.069 | 1.058 |
| | RGA | 1.100 | 1.095 | 1.072 | 1.079 | 1.100 | 1.100 |
| | DE | 1.100 | 1.094 | 1.074 | 1.076 | 1.100 | 1.100 |
| Case 2 | EP [11] | 1.099 | 1.071 | 1.023 | 1.006 | 1.070 | 1.062 |
| | RGA | 1.1000 | 1.100 | 1.063 | 1.068 | 1.097 | 1.100 |
| | DE | 1.1000 | 1.086 | 1.063 | 1.066 | 1.100 | 1.100 |

TABLE V
TRANSFORMER TAP SETTINGS

| Branch | | (6,9) | (6,10) | (4,12) | (28,27) |
|---|---|---|---|---|---|
| Case 1 | EP [11] | 0.981 | 1.042 | 1.029 | 1.037 |
| | RGA | 1.0136 | 0.9537 | 0.9895 | 0.9547 |
| | DE | 1.0331 | 0.9500 | 0.9947 | 0.9671 |
| Case 2 | EP [11] | 0.993 | 1.028 | 1.015 | 0.986 |
| | RGA | 0.9640 | 0.9778 | 0.9790 | 1.0009 |
| | DE | 0.9686 | 0.9500 | 0.9686 | 0.9923 |

TABLE VI
VAR SOURCE INSTALLMENTS

| Bus | | 6 | 17 | 18 | 27 |
|---|---|---|---|---|---|
| Case 1 | EP [11] | 0 | 0 | 0 | 0 |
| | RGA | 0 | 0 | 0 | 0 |
| | DE | 0 | 0 | 0 | 0 |
| Case 2 | EP [11] | 0.045 | 0.285 | 0.100 | 0.215 |
| | RGA | 0.360000 | 0.235836 | 0.143616 | 0.312872 |
| | DE | 0.360000 | 0.248054 | 0.163124 | 0.316619 |

TABLE VII
GENERATIONS AND POWER LOSSES

| | | $P_g$ | $Q_g$ | $P_{loss}$ | $Q_{loss}$ |
|---|---|---|---|---|---|
| Case 1 | EP [11] | 2.88362 | 0.87346 | 0.04963 | -0.38527 |
| | RGA | 2.88099 | 1.04923 | 0.046987 | 0.020225 |
| | DE | 2.88096 | 1.05594 | 0.046835 | 0.026935 |
| Case 2 | EP [11] | 5.88267 | 2.20040 | 0.21468 | 0.37715 |
| | RGA | 5.86938 | 2.10492 | 0.201383 | 0.866239 |
| | DE | 5.86906 | 2.06498 | 0.201062 | 0.861781 |

The real power savings, annual cost savings and the total costs are calculated as,

$$P_{loss}\% = \frac{P_{loss}^{init} - P_{loss}^{opt}}{P_{loss}^{init}} \times 100\ \%$$

$$W_c^{save} = hd_l\left(P_{loss,l}^{init} - P_{loss,l}^{opt}\right)$$

$$Fc = I_C + W_c$$

Table VIII gives the cost comparison. From the comparison, the DE gives better results than RGA and EP for RPP problem. Fig. 1 shows the convergence rate of the DE algorithm.

TABLE VIII
COST COMPARISON

| | | $W_C^{save}$($) | $W_C^{save(\$)}$ | $F_C$($) |
|---|---|---|---|---|
| Case 1 | EP [11] | 10.47 | 305373.6 | $2.4616*10^6$ |
| | RGA | 15.25 | 444289.7 | $2.4696*10^6$ |
| | DE | 15.52 | 452278.8 | $2.4616*10^6$ |
| Case 2 | EP [11] | 17.17 | 2338446.96 | $1.3833*10^7$ |
| | RGA | 22.29 | 3037337.28 | $1.3746*10^7$ |
| | DE | 22.42 | 3054209.04 | $1.3835*10^7$ |

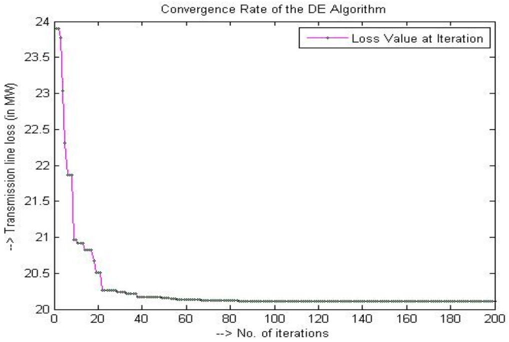


Figure 1. Convergence rate of the DE algorithm

## VI. CONCLUSIONS

The DE approach has been developed for solving the RPP problem in large-scale power systems. The application studies on the IEEE 30-bus system show that DE gives better results and always leads to the global optimum points of the multi-objective RPP problem, compared to RGA and EP. By the DE approach, more savings on the energy and installment costs are achieved and the violations of the voltage and reactive power limits are eliminated.

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