Determination of the Optimal Design of Three-Ring Concentric Circular Antenna Array Using Evolutionary Optimization Techniques

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***Abstract*—In this paper the optimal design for maximum sidelobe level (SLL) reduction of three-ring concentric circular antenna array (CCAA) is determined using a novel Particle Swarm Optimization (PSO) technique namely Craziness based Particle Swarm Optimization (CRPSO). Real coded Genetic Algorithm (RGA) is adopted for the comparative optimization. The present text assumes non-uniformly excited array and a design goal of maximizing SLL reduction using the above two evolutionary optimization techniques. Among all the designs, the three-ring structure containing (N_1=4, N_2=6, N_3=8) elements proves to be the optimal design owing to the highest SLL reduction achieved by each technique. CRPSO yields grand minimum SLL (-16.3 dB) for the above optimal set.**

***Index Terms*—Concentric Circular Antenna Array, Non-uniform Excitation, Sidelobe Level, Genetic Algorithm, Particle Swarm Optimization**

I. Introduction

An antenna array consists of multiple stationary antenna elements, which are often fed coherently. Recently, varied applications of antenna array have been suggested to improve the performance of mobile and wireless communication systems through efficient spectrum utilization, increasing channel capacity, extending coverage area, tailoring beam shape etc. [1]. However, arbitrary array design may lead to increment in pollution of the electromagnetic environment and more importantly, wastage of precious power, which may prove fatal for power-limited battery-driven wireless devices. This explains the presence of abundant open technical literatures [2-6], bearing a common target - bridging the gap between desired radiation pattern having nil SLL with what is practically achievable. The primary method in all these research works is improvement of array pattern by manipulating the structural geometry to suppress the SLL while preserving the gain of the main beam. The goal in such antenna array geometry synthesis techniques is to determine the physical layout of the array that produces the radiation pattern closest to the desired pattern. As the shape of the desired pattern can vary widely depending on the application, many synthesis methods coexist.

Among the different types of antenna arrays CCAA [2, 3, 5] have become most popular in mobile and wireless communications. This very fact has inspired the design of CCAA and evaluation of the performance of corresponding antenna arrays. In this paper optimization of CCAA design having a uniform element separation and a non-uniform excitation is performed with the help of evolutionary optimization techniques.

Contribution of the paper is twofold. Firstly, the outcome of non-uniform excitation in various CCAA design structures is examined to find the best possible design structure by two evolutionary techniques, RGA and CRPSO **[7].** Secondly, regarding the comparative effectiveness of the techniques, the newly proposed CRPSO technique proves to be the best in attaining minimum SLL, reduction of major lobe beamwidth and hence minimum "Misfitness" objective function values in the optimization of various CCAA design problems.

The rest of the paper is arranged as follows: in section II, the general design equations for the non-uniformly excited CCAA are stated. Then, in section III, brief introductions for the RGA and CRPSO are presented. Numerical results are presented in section IV. Finally the paper concludes with a summary of the work in section V.

II. Design Equation

Geometrical configuration is a key factor in the design process of an antenna array. For CCAA, the elements are arranged in such a way that all antenna elements are placed in multiple concentric circular rings, which differ in radii and in number of elements. Fig. 1 shows the general configuration of CCAA with M concentric circular rings, where the m^{th} (m = 1, 2,…, M) ring has a radius r_m and the corresponding number of elements is N_m. If all the elements (in all the rings) are assumed to be isotopic sources, then the radiation pattern of this array can be written in terms of its array factor only.

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Referring to Fig.1, the array factor, $AF(\theta, I)$ for the CCAA in *y-z* plane may be written as (1):

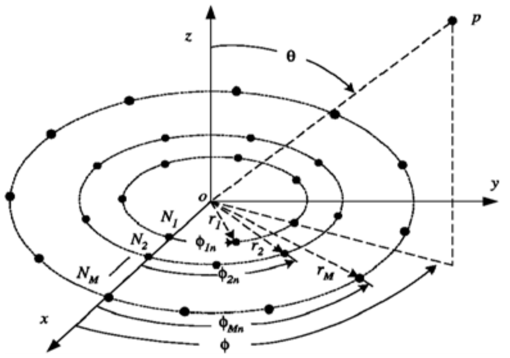

Figure 1. Concentric Circular Antenna Array.

$$AF(\theta, I) = \sum_{m=1}^{M}\sum_{i=1}^{N_m} I_{mi} \exp\left[j\left(Kr_m \sin\theta \cos(\theta - \phi_{mi}) + \alpha_{mi}\right)\right] \quad (1)$$

where I_{mi} denotes current excitation of the i^{th} element of the m^{th} ring, $K = 2\pi/\lambda$; λ being the signal wave-length, and θ and ϕ symbolize the zenith angle from the positive *z* axis and the azimuth angle from the positive *x* axis to the orthogonal projection of the observation point respectively. It may be noted that if the elevation angle is assumed to be 90 degree i.e. $\theta = 90^0$ then (1) may be written as a periodic function of ϕ with a period of 2π radian. The angle ϕ_{mi} is nothing but element to element angular separation measured from the positive x-axis. As the elements in each ring are assumed to be uniformly distributed, we have.

$$\phi_{mi} = 2\pi\left(\frac{i-1}{N_m}\right); \qquad m = 1,\cdots,M; \quad i = 1,\cdots,N_m \qquad (2)$$

The residual phase term α_{mi} is a function of angular separation ϕ_{mi} and ring radii r_m.

$$\alpha_{mi} = -Kr_m \cos(\theta_0 - \phi_{mi}); \qquad m = 1,\cdots,M; \quad i = 1,\cdots,N_m \quad (3)$$

where θ_0 is the value of θ where peak of the main lobe is obtained.

After defining the array factor, the next step in the design process is to formulate the objective function which is to be minimized. The objective function “Misfitness” (MF) may be written as (4):

$$MF = W_{F1} \times \frac{\left|AF(\theta_{msl1}, I_{mi}) + AF(\theta_{msl2}, I_{mi})\right|}{\left|AF(\theta_0, I_{mi})\right|} + W_{F2} \times \left(BWFN_{computed} - BWFN(I_{mi} = 1)\right) \qquad (4)$$

BWFN is an abbreviated form of first null beamwidth, or, in simple terms, angular width between the first nulls on either side of the main beam. MF is computed only if $BWFN_{computed} < BWFN(I_{mi} = 1)$ and corresponding solution of current excitation weights is retained in the active population otherwise not retained. W_{F1} and W_{F2} are the weighting factors. θ_0 is the angle where highest maximum of central lobe is attained in $\theta \in [-\pi, \pi]$. θ_{msl1} is the angle where the maximum sidelobe $(AF(\theta_{msl1}, I_{mi}))$ is attained in the lower band and θ_{msl2} is the angle where the maximum sidelobe $(AF(\theta_{msl1}, I_{mi}))$ is attained in the upper band. W_{F1} and W_{F2} are so chosen that optimization of SLL remains more dominant than optimization of $BWFN_{computed}$ and MF never becomes negative. In (4) the two beamwidths, $BWFN_{computed}$ and $BWFN(I_{mi} = 1)$ basically refer to the computed first null beamwidth in radian for the non-uniform excitation case and for uniform excitation respectively. Minimization of MF means maximum reductions of SLL both in lower and upper bands and lesser $BWFN_{computed}$ as compared to $BWFN(I_{mi} = 1)$. The evolutionary optimization techniques employed for optimizing the current excitation weights resulting in the minimization of MF and hence reduction in both SLL and *BWFN* are described in the next section.

III. Evolutionary Techniques Employed

A. *Real Coded Genetic Algorithm (RGA)*

GA is mainly a probabilistic search technique, based on the principles of natural selection and evolution. At each generation it maintains a population of individuals where each individual is a coded form of a possible solution of the problem at hand and called chromosome. Chromosomes are constructed over some particular alphabet, e.g., the binary alphabet {0, 1}, so that chromosomes’ values are uniquely mapped onto the decision variable domain. Each chromosome is evaluated by a function known as fitness function, which is usually the cost function or the objective function of the corresponding optimization problem.

Steps of RGA as implemented for optimization of current excitations are:

- Initialization of real chromosome strings of n_p population, each consisting of a set of excitations. Size of the set depends on the number of excitation elements in a particular CCAA design.
- Decoding of strings and evaluation of MF of each string.
- Selection of elite strings in order of increasing MF values from the minimum value.
- Copying of the elite strings over the non-selected strings.
- Crossover and mutation to generate off-springs.
- Genetic cycle updating.
- The iteration stops when the maximum number of cycles is reached. The grand minimum MF and its corresponding chromosome string or the desired solution are finally obtained.

B. Particle Swarm Optimization (PSO)

PSO is a flexible, robust population-based stochastic search/optimization technique with implicit parallelism, which can easily handle with non-differential objective functions, unlike traditional optimization methods. PSO is less susceptible to getting trapped on local optima unlike GA, Simulated Annealing, etc. Eberhart and Shi [8] developed PSO concept similar to the behavior of a swarm of birds. PSO is developed through simulation of bird flocking in multidimensional space. Bird flocking optimizes a certain objective function. Each particle knows its best value so far (pbest). This information corresponds to personal experiences of each particle. Moreover, each particle knows the best value so far in the group (gbest) among pbests. Namely, each particle tries to modify its position using the following information:

• The distance between the current position and pbest.
• The distance between the current position and gbest.

Mathematically, velocities of the particles are modified according to the following equation:

$$V_i^{(k+1)} = w * V_i^k + C_1 * rand_1 * \left(pbest_i - S_i^k\right) + C_2 * rand_2 * \left(gbest - S_i^k\right) \quad (5)$$

where V_i^k is the velocity of i[th] particle at k[th] iteration; w is the weighting function; C_j is the weighting factor; $rand_i$ is the random number between 0 and 1; S_i^k is the current position of particle i at iteration k; $pbest_i$ is the personal best of particle *i*; gbest is the group best among all pbests for the group. The searching point in the solution space can be modified by the following equation:

$$S_i^{(k+1)} = S_i^k + V_i^{(k+1)} \quad (6)$$

The first term of (5) is the previous velocity of the particle. The second and third terms are used to change the velocity of the particle. Without the second and third terms, the particle will keep on ''flying'' in the same direction until it hits the boundary. Namely, it corresponds to a kind of inertia and tries to explore new areas. The values of w, C_1 and C_2 are given in the next section.

C. Craziness based Particle Swarm Optimization (CRPSO)

The global search ability of traditional PSO is achieved with the help of the following modifications. This modified PSO is termed as CRPSO.

The velocity in this case can be expressed as follows [7]:

$$V_i^{k+1} = r_2 * sign(r_3) * V_i^k + (1-r_2) * C_1 * r_1 * \left\{pbest_i - S_i^k\right\} + (1-r_2) * C_2 * (1-r_1) * \left\{gbest - S_i^k\right\} \quad (7)$$

where r_1, r_2 and r_3 are the random parameters uniformly taken from the interval [0, 1] and $sign(r_3)$ is a function defined as:

$$sign(r_3) = -1 \quad \text{when } r_3 \le 0.05; \qquad = 1 \quad \text{when } r_3 > 0.05 \quad (8)$$

The two random parameters $rand_1$ and $rand_2$ of (5) are not independent. If both are large, both the personal and social experiences are over used and the particle is driven too far away from the local optimum. If both are small, both the personal and social experiences are not used fully and the convergence speed of the technique is reduced. So, instead of taking independent $rand_1$ and $rand_2$, one single random number r_1 is chosen so that when r_1 is large, $(1-r_1)$ is small and vice versa. Moreover, to control the balance of global and local searches, another random parameter r_2 is introduced. For birds flocking for food, there could be some rare cases that after the position of the particle is changed according to (6), a bird may not, due to inertia, fly toward a region at which it thinks is most promising for food. Instead, it may be leading toward a region which is in opposite direction of what it should fly in order to reach the expected promising regions. So, in the step that follows, the direction of the bird's velocity should be reversed in order for it to fly back into promising region. $sign(r_3)$ is introduced for this purpose. In birds' flocking or fish schooling, a bird or a fish often changes directions suddenly. This is described using a ''craziness'' factor and is modeled in the technique by using a craziness variable. A craziness operator is introduced in the proposed technique to ensure that the particle would have a predefined craziness probability to maintain the diversity of the particles. Consequently, before updating its position the velocity of the particle is crazed by

$$V_i^{k+1} = V_i^{k+1} + P(r_4) * sign(r_4) * v_i^{craziness} \quad (9)$$

where r_4 is a random parameter which is chosen uniformly within the interval [0, 1];

$v_i^{craziness}$ is a random parameter which is uniformly chosen from the interval $\left[v_i^{min}, v_i^{max}\right]$; and $P(r_4)$ and $sign(r_4)$ are defined respectively as

$$P(r_4) = 1 \quad \text{when } r_4 \le P_{cr}; \qquad = 0 \quad \text{when } r_4 > P_{cr} \quad (10)$$

$$sign(r_4) = -1 \quad \text{when } r_4 \le 0.5; \qquad = 1 \quad \text{when } r_4 \le 0.5 \quad (11)$$

where P_{cr} is a predefined probability of craziness and iter means iteration cycle number.

IV. Experimental Results

This section gives the experimental results for various CCAA designs obtained by RGA and CRPSO techniques. For each optimization technique ten three-ring (M=3) CCAA structures are assumed, each maintaining a fixed spacing between the elements in each ring (inter-element spacing for: first ring=0.55λ, second ring=0.606λ and third ring=0.75λ). These spacings are the means of the values determined for the ten structures for non-uniform spacing and non-uniform excitations in each ring using 25 trial generalized optimization runs for each structure. This generalized optimization is beyond the scope of this paper. For all sets of experiments, the number of elements of the inner most circle is N_1, for outermost circle is N_3, whereas the middle circle consist of N_2 number of elements. For all the cases, $\theta_0 = 0^0$ is considered so that the centre of the main lobe in radiation patterns of CCAA starts from the origin. After experimentation, best proven values of W_{F1} and W_{F2} are fixed as 18 and 1 respectively.

The parameters for the RGA are set after many trial runs. It is found that the best results are obtained for an initial population of 120 chromosomes. Each chromosome consists of a set of 8 bit coded excitation values. Maximum number of generations, N_m is limited to 800. For selection operation, the method of natural selection is chosen with a selection probability of 0.3. Crossover is randomly selected dual points. Crossover ratio is 0.8. Mutation probability is 0.004.

Since PSO techniques are sometimes quite sensitive to certain parameters, the simulation parameters should be carefully chosen. Best chosen maximum population pool size, n_p =120, maximum iteration cycles for optimization, N_m = 100 (CRPSO). Lesser number of cycles is found to be sufficient for the convergences of PSOs, since PSOs' convergence rates are higher than RGA's convergence rate.

CRPSO with $C_1 = C_2 = 1.5$ is experimentally found to be very effective. The predefined probability of craziness is introduced to maintain the diversity of the particles. P_{cr} = 0.3 gives the best results after several experimentation. $v_i^{craziness}$ fixed as 0.01, gives the best results. Results obtained with this technique prove to be the best compared to other two techniques considered. This novel technique has very rapidly converged to the correct optimal solution unlike RGA. Each of RGA and CRPSO techniques generates a set of normalized non-uniform current excitation weights for all sets of CCAA. I_{mi} =1 corresponds to uniform current excitation. Sets of three-ring CCAA (N_1, N_2, N_3) designs considered are (2,4,6), (3,5,7), (4,6,8), (5,7,9), (6,8,10), (7,9,11), (8,10,12), (9,11,13), (10,12,13), (11,13,15). Partial results for RGA and CRPSO are shown in Tables II-III. Table I depicts SLL values and *BWFN* values for all corresponding CCAA structures but uniformly excited.

A. Analysis of Radiation Patterns of CCAA Sets and Optimal CCAA

Fig. 2 shows the radiation patterns for a uniformly excited CCAA having different number of elements, with fixed sp**acing of $\lambda/2$ between elements and fo**r non-uniformly and optimally excited CCAA, using RGA and CRPSO optimization techniques respectively. From the figure it is clearly visible that the SLL reduction is marginal for the uniformly excited set, although the number of elements is the same.

Fig. 2 depicts the substantial reductions in SLL with non-uniform optimal current excitations. As seen from the Tables II-III, SLL reduces to -14.03dB (RGA) and -16.3dB (grand highest SLL reduction as determined by CRPSO) for the CCAA set having N_1=4, N_2=6, N_3=8. This set yields maximum SLL reductions for both techniques among all the sets.

TABLE I.

CURRENT EXCITATION WEIGHTS, SLL AND BWFN FOR UNIFORMLY EXCITED CCAA

Set No.	No. of elements in each rings (N_1,N_2,N_3)	Current excitation weights for the array elements (I_{11}, I_{12} ,... I_{mi})	SLL (dB)	BWFN (deg)
I	2, 4, 6	I_{mi} =1	-6.28	128.4
II	3, 5, 7	I_{mi} =1	-6.89	107.2
III	4, 6, 8	I_{mi} =1	-5.6	90.3
IV	5, 7, 9	I_{mi} =1	-5.6	78.2
V	6, 8, 10	I_{mi} =1	-5.17	68.4
VI	7, 9, 11	I_{mi} =1	-5.0	61.0
VII	8, 10, 12	I_{mi} =1	-4.78	54.8
VIII	9, 11, 13	I_{mi} =1	-4.64	50.0
IX	10, 12, 14	I_{mi} =1	-4.53	46.0

TABLE II.

CURRENT EXCITATION WEIGHTS, SLL AND BWFN FOR NON-UNIFORMLY EXCITED CCAA USING RGA

Set No.	Current excitation weights for the array elements (I_{11}, I_{12} ,.... I_{mi})	SLL (dB)	BWFN (deg)
III	0.3773 0.9491 0.3830 0.7861 0.5661 0.6932 0.9638 0.6275 0.5465 0.9349 0.4878 0.7220 0.5123 0.2850 0.6041 0.7300 0.5016 0.2799	-14.03	77.1
V	0.5513 0.4810 0.6504 0.5254 0.7093 0.9878 0.9240 0.0206 0.7129 0.9853 0.8481 0.0006 0.8226 0.9933 0.4945 0.7770 0.6438 0.4928 0.6184 0.4075 0.9723 0.8552 0.4231 0.5006	-12.65	59.6
VII	0.7507 0.3438 0.7676 0.9471 0.8357 0.4322 0.8105 0.9816 0.4392 0.1996 0.2429 0.8479 0.5903 0.6827 0.0409 0.0649 0.9464 0.5148 0.5861 0.0744 0.9357 0.3858 0.4818 0.4177 0.2614 0.5137 0.9845 0.6134 0.3000 0.5170	-13.12	51.8

BWFN values become narrower for non-uniform optimal current excitation weights as compared to the ordinary uniform excitation in all cases. For the CCAA set having N_1=4, N_2=6, N_3=8, the BWFN values are 77.1^0(RGA) and 76.5^0 (CRPSO) against 90.3^0 for uniformly excited CCAA having the same number of elements. So, these techniques yield maximum reductions of BWFN for this CCAA. Therefore the above set is the optimal CCAA (shown as shaded rows in the Tables II-III).

B. Comparative effectiveness and convergence profiles of RGA and CRPSO

The minimum MF values against number of iteration cycles are recorded to get the convergence profile of each technique. Figures 3-4 portray the convergence profiles of minimum MF of RGA (1.57) and CRPSO (1.27) respectively. From these figures it is clear that CRPSO converges faster (60 cycles) than RGA (250 cycles). Considering optimization cycles the execution time for 4-6-8 CCAA design (Set number III) CRPSO (2.202 minute) is the faster than RGA (9.325 minute). RGA yields suboptimal higher values of MF . CRPSO yields true optimal (least) MF consistently in all cases, though CRPSO converges to optimal solution with some small oscillations. These small oscillations are due to the introduction of craziness in the velocity, which in turn helps to determine the true global optimal solutions in the neighborhood of multiple suboptimal minimums. The oscillations due to craziness can be minimized by reducing $v^{craziness}$ without compromising much of optimality. With a view to the above facts, it may finally be inferred that CRPSO yields true optimization. The programming has been written in Matlab language using MATLAB 7.5 on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

TABLE III.

CURRENT EXCITATION WEIGHTS, SLL AND BWFN FOR NON-UNIFORMLY EXCITED CCAA USING CRPSO

Set No.	Current excitation weights for the array elements ($I_{11}, I_{12}, ... I_{mi}$)	SLL (dB)	BW FN (deg)
III	0.0906 0.6250 0.0986 0.6904 0.4267 0.4139 1.0000 0.4145 0.4393 0.9604 0.4979 0.6600 0.4866 0.2423 0.5017 0.6475 0.5020 0.2387	-16.3	76.5
V	0.6462 0.7146 0.8599 0.6331 0.5285 0.9829 0.9326 0.0000 0.8051 0.9424 0.8323 0.0000 1.0000 0.9999 0.4039 0.8987 0.8780 0.3347 0.6025 0.5059 0.8089 0.8498 0.5761 0.6353	-12.86	60.5
VII	0.6779 0.3910 0.6668 1.0000 1.0000 0.3036 0.9360 0.9757 0.5663 0.2201 0.2306 0.6152 0.6135 0.8425 0.1443 0.1248 0.6574 0.5085 0.4546 0.2234 0.9887 0.2936 0.4100 0.4502 0.3979 0.4542 1.0000 0.3310 0.4025 0.5632	-13.64	53.8

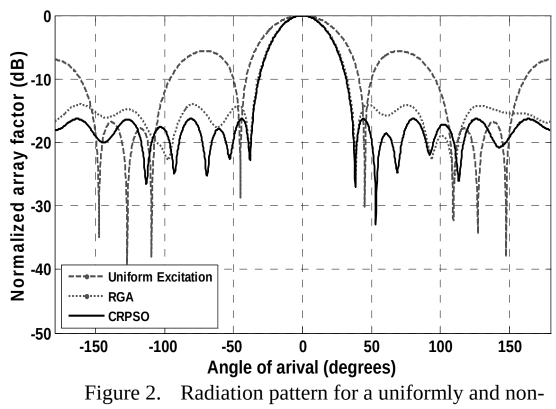

Figure 2. Radiation pattern for a uniformly and non-uniformly excited CCAA (N_1=4, N_2=6, N_3=8 elements).

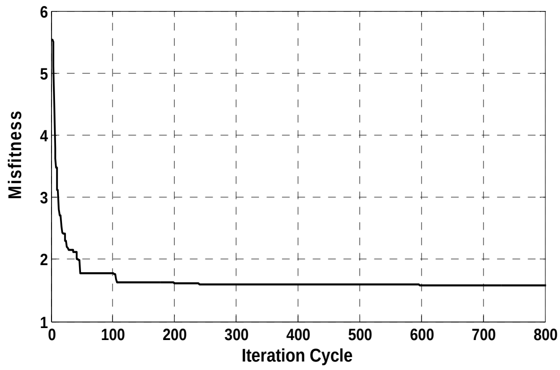

Figure 3. Convergence curve for RGA in case of non-uniformly excited CCAA (N_1=4, N_2=6, N_3=8 elements).

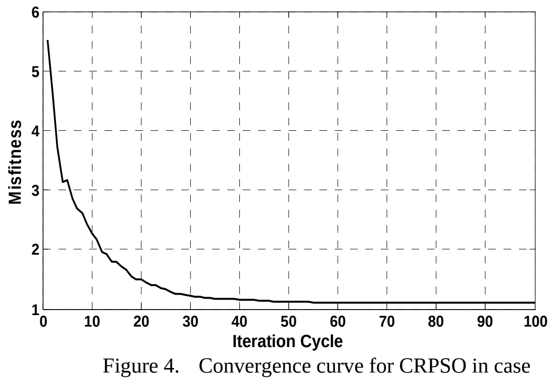

Figure 4. Convergence curve for CRPSO in case of non-uniformly excited CCAA (N_1=4, N_2=6, N_3=8 elements).

CONCLUSION

In this paper, the design of a non-uniformly excited concentric circular antenna array with uniform spacing between the elements has been described using the techniques of RGA and CRPSO. Comparing with the other techniques reported in the work, CRPSO technique proves to be moderately fast and robust technique yields true optimal excitations and global minimum values of SLL for all sets of CCAA designs. RGA is less robust and yield suboptimal results. Experimental results reveal that design of non-uniformly excited CCAA offers a considerable SLL reduction along with the reduction of BWFN with respect to corresponding uniformly excited CCAA. Contribution of the paper is twofold: first, for three techniques the CCAA having N_1=4, N_2=6, N_3=8, gives grand maximum SLL reduction compared to all other sets, which one is the optimal set among three-ring structures, and second, comparing the performance of both techniques CRPSO shows the better optimization performance compared to RGA. Thus, the newly proposed CRPSO technique is expected to be very promising evolutionary optimization technique for the global optimization of any antenna array problem.

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