SHORT PAPER
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Signal Filtering Using Discrete Wavelet Transform

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***Abstract*—Noise has been a primary deterrent in signal transmission and processing. It results in faulty information after processing the signals reducing their usability. In our work, Daubechies wavelet was used to filter out the noise from a discretely sampled signal by implementing a low pass filter. We applied wavelet transform on the input vector, threshold it, inverse transformed it to finally achieve a signal with very low noise. The motivation for this research work was derived from the quest to develop wavelet based filters for ECG signal filtering and feature extraction with better results as compared to FIR filters. The other primary aim was to reduce the cost of the equipments where such filter is being used as a component.**

***Index Terms*—wavelet, MRA, DWT, Daubechies wavelets, denoising.**

I. Introduction

Signals always have some noise associated with them, rarely do we find signals in "real-life" situations that are free from noise and can be directly employed for extracting information. Noise can result in an output which may not be intended or not the characteristic of the quantity being observed, giving rise to faults in the system of which the signal is a component. It can also cause judgmental errors if the signal is being directly observed and the impact can range from being minute in some cases to destructive in certain critical systems like ECG machines.

Hence, it is important that signals should comprise of components that are relevant to the system and be free from unwanted, random values so that the errors caused due to faulty representation of the original signal can be minimized. Most of the times the noise found in the signals is of higher frequency as compared to the signal produced by the quantity being measured or represented. It is, therefore, of utmost importance that the noise from the signal is removed to the optimal extent.

As explained above, the problem of noise in signals is not new. Various solutions have been proposed and are currently being employed in a number of systems. The solutions range from hardware implemented active filters to Finite impulse response (FIR) filters. The problem with most of the current filters in place is that they do not give sufficiently good output results or alter the original signal itself, for e.g. the FIR filters effect the signal because of the sinc function. Although such effects can be removed by the application of appropriate processes, the involved overhead results in increase in the cost of the system. Also, in some cases, the desired output levels are still not observed.

In our approach, wavelet transform and inverse wavelet transform were used. Since there are a huge number of wavelet families having several different wavelets having high number of vanishing moments and capable of representing complex polynomials, it was not difficult to find a wavelet which was similar to the signal being processed. When transformed with the similar wavelet, the disturbances caused in the original signal were minimized which reduced the overhead.

The key points in this approach were analyzing the signal to find a suitable wavelet, applying the transform in algebraic form, performing threshold operations and inverse transforming it. The main limitation was that the hard thresholding scheme that had been followed in our work, although gave better results but also introduced about 1% random values of the size of the input vector which were within 4-5 % of the correct value.

II. Wavelets

Wavelets are mathematical functions with oscillatory nature similar to sinusoidal waves with the difference being that they are of "finite oscillatory nature". Essentially a finite length, decaying waveform, when scaled and translated results in what is called a "daughter wavelet" of the original "mother wavelet". Hence different scaling and translation variables result in a different daughter wavelet from a single mother wavelet.

Wavelet transforms are classified as Continuous wavelet transforms (CWT) and Discrete wavelet transforms (DWT). The finite oscillatory nature of the wavelets makes them extremely useful in real life situations in which signals are not stationary. While Fourier transform of a signal only offers frequency resolution, wavelet transforms offer "variable time-frequency" resolution which is the hallmark of wavelet transforms.

A wavelet transform decomposes a signal into basis functions which are known as wavelets. Wavelet transform is calculated separately for different segments of the time-domain signal at different frequencies resulting in Multi-resolution analysis or MRA. It is designed in such a way that the product of time resolution and frequency resolution is constant. Therefore it gives good time resolution and poor frequency resolution at high frequencies whereas good frequency resolution and poor time resolution at low frequencies. This feature of MRA makes it excellent for signals having high frequency components for short durations and low frequency components for long duration .e.g. noise in signals, images , video frames etc.

A.. Discrete Wavelet Transform

A wavelet transform in which the wavelets are discretely sampled are known as *discrete wavelet transform*. The DWT gives a multi-resolution description of a signal which is very useful in analyzing "real-world" signals. Essentially, a discrete multi-resolution description of a continuous-time signal is obtained by a DWT. It converts a series a_0, a_1, a_2……. a_m into one low pass coefficient series known as “approximation” and one high pass coefficient series known as “detail”. Length of each series is m/2.In real life situations, such transformation is applied recursively on the low-pass series until the desired number of iterations is reached.

Some examples of discrete wavelets are the Haar wavelets, Daubechies wavelets, symmlets etc. For any input comprising of 2^n numbers, the Haar wavelet transform simply pairs up input values, storing the difference and passing the sum. This process is recursive, pairing up the sums to provide the next scale: finally resulting in 2^{n-1} differences and one final sum and this is done in O(n) time i.e. linear time.

The function is not continuous and hence not differentiable. Daubechies wavelets are families of wavelets whose inverse wavelet transforms are adjoint of the wavelet transform i.e. they are orthogonal. They have maximal number of vanishing moments and hence they can represent higher degree polynomial functions. With each wavelet type of this class, there is a scaling function known as “father wavelet” that generates an orthogonal multi-resolution analysis .Daubechies orthogonal wavelets D2-D20 (even index numbers only) are commonly used. The numbers associated with the name refers to the number ‘N’ of coefficients. Each wavelet has vanishing moments equal to half the number of coefficients. For example, D2 which is the Haar wavelet has one vanishing moment, D4 has two, etc. The number of vanishing moments is what decides the wavelet's ability to represent a signal. For example, D2, with one moment, easily encodes polynomials of one coefficient, or constant signal components. D4 encodes polynomials with two coefficients, i.e. constant and linear signal components etc. The wavelet transform using Daubechies wavelets result in progressively finer discrete samplings using recurrence relations. Every resolution scale is double that of the previous scale. Daubechies derived a family of wavelets, the first of which is the Haar wavelet. Since then interest in this field has shot up and many variations of Daubechies original wavelets have been developed. The discrete wavelet transform has applications ranging from data compression to signal coding. In our research work, Daubechies wavelet was used to filter a noisy signal to extract information from the signal.

III. Signal Filtering

As is the case with most of the signals in real life, they are always accompanied with noise which may be random, Gaussian white noise etc. Till this noise is present with the signal, the received signal may be of very little use. Noise makes the process of information extraction from the signal a difficult task and results to incorrect output. Signal filtering is a process which can be thought as a “pre–processing step” for information extraction from the signal. Generally noise is a low amplitude high frequency signal imposed on the higher amplitude lower frequency signal.

Let S(n) be the original signal with no noise, V(n) be the high frequency noise added to the signal before it is received for analysis or information extraction.

The signal received be represented by VS(n) as follows:

$$VS(n) = S(n) + V(n)$$

The purpose of the procedure of denoising is to extract S(n) from VS(n) so that it can be used for intended purposes.

Noise added to the signal is higher in frequency as compared to the original signal. Hence if we can remove the high frequency components of the signal, we would be able to separate the noise. This can be achieved by passing VS(n) through a low pass filter which will filter out the high frequency components and give the output which would be approximately close to S(n).

IV. Signal Filtering Using Wavelet Transforms

In our research work the properties of Discrete wavelet transforms were employed to recover a signal from the signal with noise. The process of filtering can be broken into further steps which are: Analysis, Applying wavelet transform, Threshold and Inverse wavelet transform. In some cases first two steps can be combined.

A. Analysis Step

Selecting an appropriate wavelet was a very important task in this step. The wavelet chosen should be similar to the signal that has to be filtered to give the best possible results. This “similarity” can be decided on the basis of the cross- correlation between the two functions. We had preferred the Daubechies family of wavelets because of their high number of vanishing moments making them capable of representing complex high degree polynomials. The result of our simulations showed that D4 wavelet provided sufficiently good signal output. The wavelet has 4 constants (c_0, c_1, c_2, c_3) related to it whose values have been derived by Daubechies in her research work. These are normalized according to the need of the application where they are being used. The matrix operations shown in figure below reveal that a vector with 2^n elements is numerically transformed to 2 vectors both having 2^{n-1} elements each. One of the vectors has the “approximate” or smooth coefficients and the other has detail coefficients. The vector with smooth coefficients served as input for the next iteration of the same procedure. This procedure was carried out till the size of the smooth coefficients vector was greater than or equal to 4.

B. *Threshold Step*

After applying the selected wavelet transform to the input vector we obtained a numerically transformed vector which had the detail coefficients that are carried from one level to the next as it is and the final left approximation values. To denoise the signal, the detail coefficients were made '0' after applying the transform. Then, the size of the input vector, which had sampled values of the signal, was known and it was also known that each time the size of the resultant vector had been reduced to half the original size.

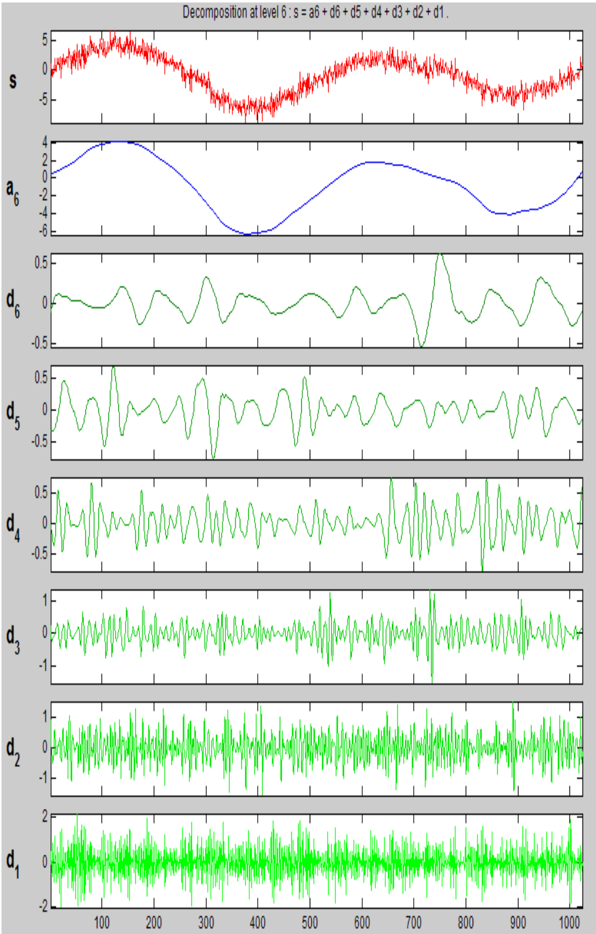

Figure 1. Our signal's' a sine wave with noise, After applying the wavelet transform to the signal: a6 is the approximation at 6th level, d1-d6 are the detail coefficients at respective levels which are set to '0'in the threshold step.

C. *Inverse Wavelet Transform*

After applying the wavelet transform and threshold procedures, the inverse wavelet transform was applied. The output of this step, as seen in the figure, was the original signal with very less noise as compared with the earlier signal. All the detail coefficients up to level 6 had been set to zero in the threshold step to obtain this output.

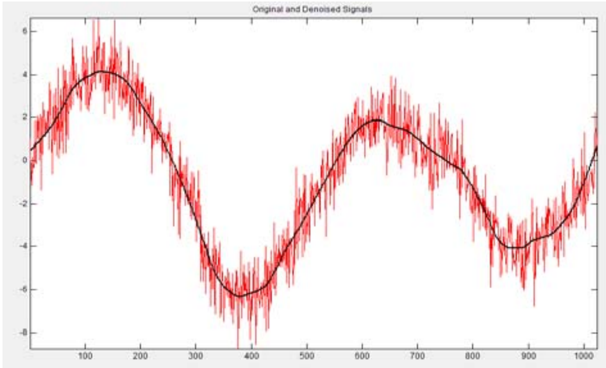

Figure2.Comparison between the original and filtered signal.

V. Conclusion

Wavelets with their "variable time frequency resolution" and properties such as MRA and high number of vanishing moments provide an effective way to analyze a signal. The process of signal filtering can be performed in quick time following this approach. The simulations that had been performed reveal that wavelets can be used to separate different frequency components of the signal efficiently. After separating the signal into components, the unwanted signal components can be removed by setting the detail coefficients related to those particular components to zero. The inverse transform can then be applied on the semi processed vector to get back the original signal which is free from noise components. It was observed that there is a close similarity between original signal without noise and the signal obtained after the filtering procedure

This process, as shown in our research, can be used to filter out noise from signals like ECG signal etc. and the algorithm can easily be implemented in C language resulting in direct practical implementations. This process can also be used for ECG feature extraction with suitable modification in the threshold step in the future.

References

[1] Matthias Schwab , "Transformation of Discrete Arrays by Daubechies Wavelets", Stanford Exploration Project.
[2] The Wavelet Digest, www.wavelet.org .
[3] S.G. Mallat, A Wavelet Tour of Signal Processing Second Edition (Academic Press, New York, 1999).
[4] http://www.bearcave.com/misl/misl_tech/wavelets/ index.html350.
[5] I. Daubechies, Ten Lectures on Wavelets (PA SIAM, Philadelphia, 1992).
[6] Kaiser, G., A Friendly Guide To Wavelets. Boston: Birkhäuser, 1994.
[7] Ali N. Akansu, "An Efficient QMF-Wavelet Structure", 1st Wavelet Symposium, New Jersey Institute Of technology, April 1990.
[8] http://www.amara.com/current/wavelet.html
[9] Brani Vidakovic, Peter Mueller, "Wavelets for Kids", A tutorial introduction by Duke University.

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