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Fuzzy Reliability Measures of Fuzzy Probabilistic Semi-Markov Model

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***Abstract*— Software systems are becoming increasingly more complex and testing it to a reliable system requires a great deal of effort. Stochastically testing promises a solution to this increased testing burden and gives the opportunity to analyze reliability, mean time to failure etc. Today there is hundreds of reliability models with more models developed every year. Still, there exist no models that can be applied for the cases that are uncertain /imprecise. In this paper we present the feasibility of applying fuzzy technique testing based on fuzzy probabilistic semi-Markov chain usage model to web based application and explore the fuzzy reliability for the systems that are based on uncertainty/vagueness. The definitions and results for the fuzzy model are provided by means of the fuzzy probabilities and are modeled by triangular fuzzy numbers.**

***Index Terms*— Fuzzy Probability, Triangular Fuzzy Number, Fuzzy Probabilistic semi-Markov Model, Fuzzy States, Fuzzy Reliability.**

I. Introduction

Considerable work was done on building a systematic theory of reliability based on probability theory, where the probability of a system is expressed in terms of the statistical information of its subsystem. There are two fundamental assumptions on conventional reliability theory

- Binary State Assumption: the system is precisely defined as functioning or failing.
- Probability State Assumption: the system behavior is fully characterized in the context of probability measures.

For many systems due to uncertainties and imprecision of data, the estimation of precise values of probabilities is very difficult. For this reason, the concept of fuzzy reliability have been introduced and formulated in the context of the possibility measures. Cai [7] presented the following two alternative assumptions:

- Fuzzy State Assumption: the meaning of the system failure cannot be precisely defined in a reasonable way. At any time the system may be thought of being in some extent, in one of two fuzzy states, fuzzy success state and fuzzy failure state.
- Possibility Assumption: the system behavior can be fully characterized in the context of possibility measures

And Cai [7] pointed out that there are various forms of fuzzy reliability theories namely PROBIST, POSBIST and PROFUST reliability theories.

Important theoretical results and applications for fuzzy reliability were found in [1 – 4, 6, 7]. We note that important theoretical results and applications for semi-Markov models were found in Cinlar [9], Iosifescu-Manu [11], Korve [17], Howard [10], McClean [14], Janssen [12], Usman Yusuf Abubakar [16], Janssen and Limnios [13] and in Limios and Oprisan [19]. The non homogeneous semi-Markov system in discrete time was examined in Vassiliou and Papadopoulou [15], and the asymptotic behavior of the same model was studied in Papadopoulou and Vassiliou [21]. As, we know that the states of a system are the main part to indicate the ability of function, we see that in many real life problems, the system states are very often fuzzy or imprecise. In order to test the performance of such systems that are based on fuzziness, we propose a method to find the fuzzy reliability of a non homogeneous fuzzy probabilistic semi-Markov model consisting of set of states whose transitions are fuzzy transitions between the states together with time of transition between the states, based on the assumption of fuzzy profust reliability theory i.e. with fuzzy states and probability assumptions through its transition fuzzy probabilities and waiting time fuzzy probabilities which are represented as triangular fuzzy numbers. The proposed method models and explores the fuzzy system reliability through transition fuzzy probabilities and waiting time fuzzy probabilities and as a consequence the fuzzy reliability is represented as a triangular fuzzy number.

The paper is constructed as follows: Section II recalls the preliminaries needed for this paper and section III gives the basic equations of a non homogeneous fuzzy probabilistic semi-Markov model with transition fuzzy probabilities. Section IV presents the method to find fuzzy system reliability based on the fuzzy sets using through the transition fuzzy probabilities and waiting time fuzzy probabilities of a non homogeneous fuzzy probabilistic semi-Markov model. Section V illustrates the above constructed model for web navigational model and finally we end up with the conclusions. All definitions and results for the fuzzy model are provided by means of the fuzzy probabilities and are modeled as a triangular fuzzy number.

II. Preliminaries

Here we recall some definitions that are used in this paper.

A. Fuzzy Number[8]

A fuzzy number is a fuzzy set with the following conditions:

– convex fuzzy set.
– normalized fuzzy set.
– it's membership function is piecewise continuous.
– It is defined in the real number.

B. Triangular Fuzzy number [8]

A fuzzy number $\overline{A}(a_1, a_2, a_3)$ with the following membership function is called the triangular fuzzy number.

$$\mu_{\overline{A}}(x) = \begin{cases} 0 \quad \text{if} \quad x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} \quad \text{if} \quad a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} \quad \text{if} \quad a_2 \le x \le a_3 \\ 0 \quad \text{if} \quad x > a_3 \end{cases}$$

In this paper, we have used the triangular fuzzy number on [0, 1].

C. Arithmetic Operations on Triangular Fuzzy Number[3]

Consider two triangular fuzzy numbers $\overline{A}$ and $\overline{B}$ parameterized by the triples (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively defined on $\Re^+$. The arithmetic operations between the triangular fuzzy numbers are defined as follows:

(1) Fuzzy Number Addition $\oplus$:

$\overline{A} \oplus \overline{B} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)$
$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(2) Fuzzy Number Subtraction $\ominus$:

$\overline{A} \ominus \overline{B} = (a_1, a_2, a_3) \ominus (b_1, b_2, b_3)$
$= (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
$1 \ominus (a_1, a_2, a_3) = (1, 1, 1) \ominus (a_1, a_2, a_3)$
$= (1 - a_3, 1 - a_2, 1 - a_1)$

(3) Fuzzy Number Multiplication $\otimes$:

$\overline{A} \otimes \overline{B} = (a_1, a_2, a_3) \otimes (b_1, b_2, b_3)$
$= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$

(4) Fuzzy Number Division $\oslash$:

$\overline{A} \oslash \overline{B} = (a_1, a_2, a_3) \oslash (b_1, b_2, b_3)$
$= (a_1 / b_1, a_2 / b_2, a_3 / b_3)$

D. Fuzzy Event [7]

A fuzzy event is a fuzzy set defined on a universe of discourse whose membership function is Borel measurable.

III. Non Homogeneous Fuzzy Probabilistic Semi-Markov Model

To reflect the available information about the 'true' probability distribution for the random experiment, a possibility measure (fuzzy number) has been introduced on the class of probability measures of the random variable. In this paper, we consider a random experiment, which has certainty in its outcomes and only have the uncertainty in the probability of the events and these uncertainties in the probabilistic usage information are represented by fuzzifing the probability values into a triangular fuzzy number on [0,1] for the system to perform its function properly. Thus, the transition probability between the states which is obtained from the probabilistic usage information becomes transition fuzzy probability and throughout the paper we mention the probabilities based on the usage as fuzzy probabilities that are represented as triangular fuzzy number [18, 20]. In this section we briefly review the main definitions and results from the theory of non homogeneous fuzzy probabilistic Markov renewal processes which are directly relevant for our purpose. We now define the non homogeneous fuzzy probabilistic semi-Markov model with fuzzy transitions.

Let (Ω, F, P) be a probability space and let E be a finite state space. On our probability space, we define two random variables:

$$X_n : \Omega \to E \qquad T_n : \Omega \to N$$

X_n represents the state at the n-th transition and T_n is the time of the n-th transition.

The process *(X, T)* is a non homogeneous fuzzy probabilistic Markov Renewal Process if $\forall i,\ j \in E$ and $\forall t \in N$, the following condition holds:

$$\overline{P}[X_{n+1} = j, T_{n+1} \le t / X_n = i, T_n = s, X_{n-1}, T_{n-1}, \cdots, X_0, T_0]$$
$$= \overline{P}[X_{n+1} = j, T_{n+1} \le t / X_n = i, T_n = s]$$

and for $j \ne i$,

$$\overline{Q_{ij}}(s,t) = \overline{P}[X_{n+1} = j, T_{n+1} \le t / X_n = i, T_n = s]$$

is the associated non homogeneous fuzzy probabilistic semi-Markov kernel $\overline{Q}$. The fuzzy probabilistic semi-Markov kernel is written again as:

$$\overline{Q_{ij}}(s,x) = \overline{P}[X_{n+1} = j, T_{n+1} \le x / X_n = i, T_n = s]$$

The second argument of $\overline{Q}$ namely x represents the duration time whereas s represents the starting time.

The fuzzy transition matrix $\overline{P}(s)$ of the non homogeneous extended fuzzy probabilistic Markov chain X_n is obtained as $\overline{\mathrm{p}}_{\mathrm{ij}}(\mathrm{s}) = \lim\limits_{\mathrm{x}\to\infty} \overline{\mathrm{Q}}_{\mathrm{ij}}(\mathrm{s},\mathrm{x}),\ \forall \mathrm{i},\ \mathrm{j} \in \mathrm{E}$

However, before the entrance to *j* the process holds for a time *'x'* in state *i*. The conditional cumulative fuzzy probabilistic distribution function of the waiting time in each state, given the state subsequently is given by

$$\overline{F_{ij}}(s,x) = \overline{P}[T_{n+1} \le x / X_n = i, X_{n+1} = j, T_n = s].$$

This fuzzy probabilistic function is obtained by

$$\overline{F}_{ij}(s,x)=\begin{cases}\dfrac{\overline{Q}_{ij}(s,x)}{\overline{p}_{ij}(s)}, & \text{if} \quad \overline{p}_{ij}(s)\neq 0\\ 1, & \text{if} \quad \overline{p}_{ij}(s)=0\end{cases}$$

And for more feasibility, it is supposed free of the time '*s*', namely $\overline{F}_{ij}(x)$. Without loss of generality, the waiting time also has a fuzzy probability density function namely $\overline{f}_{ij}(x)$ and $\overline{D}(x)=\left[\overline{f}_{ij}(x)\right]_{i,j}$ represents the duration time matrix.

Let us introduce the fuzzy probability that the process stays in state *i* for at least duration time '*x*', given state *i* entered at time '*s*':

$$\overline{H}_i(s,x)=\overline{P}\left[T_{n+1}-T_n\leq x/X_n=i,T_n=s\right]$$

Of course, $\overline{H}_i(s,x)=\sum_{j\neq i}^{n}\overline{Q}_{ij}(s,x)=\sum_{j\neq i}^{n}\overline{p}_{ij}(s)\overline{F}_{ij}(s,x)$.

Therefore the marginal cumulative fuzzy probability distribution function of the waiting time in each state depends on both times. Let us define $\overline{S}_i(s,x)=1-\overline{H}_i(s,x)$.

Let $N(t)=\max\{n: T_n\leq t\}$, $\forall t\in N$. We define the non homogeneous discrete time fuzzy probabilistic semi-Markov process $Z=(Z(t), t\in N)$ as $Z(t)=X_{N(t)}$, that represents for each waiting time, the state occupied by the process.

We define $\forall$ $i, j\in E$ and $(s, t)\in NxN$, the fuzzy probabilistic semi-Markov's interval transition fuzzy probabilities as

$$\overline{\phi}_{ij}(s,t)=\overline{P}\left[Z(t)=j/Z(s)=i\right]$$

satisfying the following system of equations:

$$\overline{\phi}_{ij}(s,t)=\overline{\delta}_{ij}\left(1-\overline{H}_i(s,t)\right)+\sum_{k\in E}\sum_{\tau=1}^{t}\overline{p}_{ik}(s)\overline{f}_{ik}(\tau)\overline{\phi}_{kj}(\tau,t),$$

where $\overline{\delta}_{ij}=\begin{cases}(0,0,0), & i\neq j\\ (1,1,1), & i=j\end{cases}$.

At this time, we explain briefly in the next section the fuzzy reliability model using non homogeneous fuzzy probabilistic semi-Markov model through its transition fuzzy probabilities and waiting time fuzzy probabilities.

IV. Fuzzy Reliability Measures Of A Fuzzy Probabilistic Semi-Markov Model

Fuzzy reliability is a concept in which fuzzy sets can capture subjective, uncertain and ambiguous information in a system. Now we present the fuzzy reliability modeling using fuzzy probabilistic semi-Markov model based on the fuzzy profust reliability theory through the transition fuzzy probabilities and waiting time fuzzy probabilities.

Consider a fuzzy probabilistic semi-Markov model $\{(S_n, T_n), n\in N\}$ consisting of 'n' states together with transition time. Let $U=\{S_1, S_2,\ldots, S_n\}$ denote the universe of discourse. On this universe we define a fuzzy success state S: $S=\{S_i,\overline{\mu}_S(S_i); i=1,2,\cdots,n\}$ and a fuzzy failure state F: $F=\{S_i,\overline{\mu}_F(S_i); i=1,2,\cdots,n\}$, where $\overline{\mu}_S(S_i)$ and $\overline{\mu}_F(S_i)$ are triangular fuzzy numbers.

A fuzzy state is just a fuzzy set and fuzzy states are defined to represent the system level of performance. When fuzziness of interest is discarded, the fuzzy success state and the fuzzy failure state become a conventional success state and failure state respectively. In the conventional reliability theory, one is interested in the event of transition from system success state to system failure state. Accordingly, we are here interested in the event, denoted by $\overline{T}_{SF}$, of transition from the fuzzy success state to the fuzzy failure state. Assuming that the universe *U* or the behavior of n system states is completely stochastically characterized in the time domain, we define

$$\overline{R}(t_0,t_0+t)=\overline{P}\begin{bmatrix}\overline{T}_{SF}\text{ does not occur in the time interval}\\ \text{starting from } t_0 \text{ to } t_0+t\end{bmatrix}$$

$\overline{R}(t_0,t_0+t)$ is referred as the fuzzy interval reliability of the system in the time interval starting from t_0 to t_0+t.

To compute the fuzzy interval reliability we must express $\overline{T}_{SF}$. Since both S and F are fuzzy states, the transitions between them are consequently fuzzy. We view $\overline{T}_{SF}$ as a fuzzy event. Apparently $\overline{T}_{SF}$ may occur only when some state transition occur among the n system states $\{S_1, S_2, \ldots, S_n\}$, so $\overline{T}_{SF}$ can be defined on the universe $U_{\overline{T}}=\{\overline{p}_{ij}(t_0), i, j=1,2,\cdots,n\}$, where $\overline{p}_{ij}(t_0)$ represents the transition fuzzy probability from S_i to S_j with membership function: $\{\overline{\mu}_{\overline{T}_{SF}}(\overline{p}_{ij}(t_0)), i, j=1,2,\cdots,n\}$

i.e. $\overline{T}_{SF}=\{\overline{p}_{ij}(t_0),\overline{\mu}_{\overline{T}_{SF}}(\overline{p}_{ij}(t_0)), i, j=1,2,\ldots,n\}$

Let $$\overline{\beta}_{F/S}(S_i)=\frac{\overline{\mu}_F(S_i)}{\overline{\mu}_F(S_i)+\overline{\mu}_S(S_i)}.$$

Then $\overline{\beta}_{F/S}(S_i)$ can be viewed as the grade of membership of S_i relative to S, to F. It is reasonable to say that the fuzzy transition from S_i to S_j makes the fuzzy transition from S to F occurs to some extent if and only if the relation $\overline{\beta}_{F/S}(S_j)>\overline{\beta}_{F/S}(S_i)$ holds. We therefore define

$$\overline{\mu}_{\overline{T}_{SF}}(\overline{p}_{ij}(t_0))=\begin{cases}\overline{\beta}_{F/S}(S_j)-\overline{\beta}_{F/S}(S_i) & \textit{when } \overline{\beta}_{F/S}(S_j)>\overline{\beta}_{F/S}(S_i)\\ 0 & \textit{when } \overline{\beta}_{F/S}(S_j)\leq\overline{\beta}_{F/S}(S_i)\end{cases}$$

Hence, the fuzzy interval reliability can be expressed as

$$\bar{R}(t_0,t_0+t)=1-\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{\begin{array}{l}\bar{\mu}_{TSF}\left(\bar{p}_{ij}(t_0)\right)\left(\begin{array}{l}\text{Fuzzy probability}\\ \text{that the system}\\ \text{holds for time } t_0+t\\ \text{in } S_i \text{ and transition}\\ \text{to } S_j \text{ occurs}\end{array}\right)\\ .\left(\begin{array}{l}\text{Fuzzy probability that the}\\ \text{system is in the state } S_i\\ \text{for duration time } t_0+t\end{array}\right)\end{array}\right\}$$

$$=1-\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{\begin{array}{l}\bar{\mu}_{TSF}\left(\bar{p}_{ij}(t_0)\right)\bar{f}_{ij}(t_0+t).\\ \bar{S}_i(t_0,t_0+t)\end{array}\right\}$$

$\bar{R}(t_0,t_0+t)$ may be further expressed as

$$\bar{R}(t_0,t_0+t)=\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{\begin{array}{l}\bar{\mu}'_{TSF}\left(\bar{p}_{ij}(t_0)\right)\bar{f}_{ij}(t_0+t).\\ \bar{S}_i(t_0,t_0+t)\end{array}\right\}$$

where $\bar{\mu}'_{TSF}\left(\bar{p}_{ij}(t_0)\right)=1-\bar{\mu}_{TSF}\left(\bar{p}_{ij}(t_0)\right)$.

When $t_0 = 0$, we have $\bar{R}(t_0,t_0+t)=\bar{R}(t)$.

$\bar{R}(t)$ is referred to as the fuzzy reliability of the system at time t. $\bar{R}(t)$ can be physically interpreted as the fuzzy probability deteriotion occurs in the time interval $(0, t)$. We may also define the stationary fuzzy reliability as follows:

$$\bar{R}^S(t)=\lim_{t_0\to\infty}\bar{R}(t_0,t_0+t)$$

where $\bar{R}^S(t)$ represent the stationary fuzzy reliability.

Now let us define other reliability measures. The system fuzzy availability at duration time t, denoted by $\bar{A}(0,t)$ or $\bar{A}(t)$, is defined as the fuzzy probability that the system remains in S at time duration t i.e.,

$$\bar{A}(t)=\bar{P}(S/t)=\sum_{i=1}^{n}\bar{\mu}_S(S_i).S_i(0,t)$$

The system fuzzy unavailability at duration time t denoted by $\bar{A}'(t)$, is defined as the fuzzy probability that the system remains in F at time duration t i.e.,

$$\bar{A}'(t)=\bar{P}(F/t)=\sum_{i=1}^{n}\bar{\mu}_F(S_i).S_i(0,t)$$

V. Illustration

In the following, we illustrate the above defined fuzzy reliability model with a real time example. Let us consider the web navigation of our website www.ssn.edu.in. The operational units are the web pages of department: S_1 as Computer science department (CS), S_2 as Information Technology department (IT) and S_3 as Electronics and Communication department (EC), which are the set of states and the associated connections, are the transitions. Since there exists uncertainties in the probabilistic usage information between the state transitions, for each transition we associate fuzzy transition defined as transition fuzzy probabilities obtained as follows: With the data extracted from the log files for the period of ten days, we find N-the total number of transitions from state i to state j and s-the number of successes among them, then the ratio for transition probability $\frac{s}{N}$. Since these values (N and s) which are extracted from the path that exists in the specified period of time are not exact, we fuzzify these values using the formula $\left[p-(1-\alpha)\sqrt{\frac{p(1-p)}{N}},\, p+(1-\alpha)\sqrt{\frac{p(1-p)}{N}},\ \text{for } \alpha\in[0,1]\right]$ to form the transition fuzzy probabilities as triangular fuzzy numbers. As the access of web increases, the data for the access will also increase. Hence we have modeled fuzzy probabilistic semi-Markov model with state space U= {CS, IT, EC} and transitions as transition fuzzy probabilities and is depicted below.

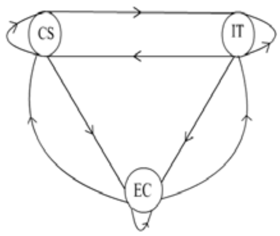

Figure 1. Transition Diagram

The fuzzy success state, fuzzy failure state and the transition fuzzy probabilities for the given state space are interpreted as follows:

S = {S_1/(0.9838,0.984,0.9841), S_2/(0.9807,0.981,0.9813), S_3/(0.9613,0.962,0.9627)}

F = {S_1/(0.0158,0.016,0.0162), S_2/(0.0187,0.019,0.0193), S_3/(0.0373,0.038,0.0387)}

$$\bar{P}=\begin{array}{c} \\ CS\\ IT\\ EC\end{array}\begin{array}{c}\begin{array}{ccc}CS & IT & EC\end{array}\\ \begin{pmatrix}(0.9838,0.984,0.9841) & (0.9858,0.987,0.9882) & (0.9868,0.989,0.9912)\\ (0.9874,0.989,0.9906) & (0.9807,0.981,0.9813) & (0.7707,0.776,0.7813)\\ (0.9932,0.996,0.9988) & (0.6524,0.662,0.6716) & (0.9613,0.962,0.9627)\end{pmatrix}\end{array}$$

The duration matrix for the given period is as follows:

$$(\bar{f}_{ij}(10))=\begin{array}{c} \\ CS\\ IT\\ EC\end{array}\begin{array}{c}\begin{array}{ccc}CS & IT & EC\end{array}\\ \begin{pmatrix}(0.978,0.981,0.984) & (0.979,0.982,0.985) & (0.897,0.9,0.903)\\ (0.887,0.89,0.893) & (0.961,0.964,0.967) & (0.838,0.841,0.844)\\ (0.963,0.966,0.969) & (0.869,0.872,0.875) & (0.777,0.78,0.783)\end{pmatrix}\end{array}$$

for $i, j = CS, IT, EC$.

Fuzzy probabilities of staying in each state for the given period are given as follows:
S_{CS} (1, 10) = (0.986, 0.989, 0.992);
S_{IT} (1, 10) = (0.979, 0.982, 0.985);
S_{EC} (1, 10) = (0.938, 0.941, 0.944)
The values of $\bar{\mu}_{TSF}\left(\bar{p}_{ij}(t_0)\right)$ are tabulated below as:

j \ i	1	2	3
1	(0,0,0)	(0.0029,0.003,0.0031)	(0.0215,0.022,0.0225)
2	(0,0,0)	(0,0,0)	(0.0186,0.019,0.0194)
3	(0,0,0)	(0,0,0)	(0,0,0)

Thus the fuzzy reliability of our website is given by
$\bar{R}(1,10) = (0.961, 0.962, 0.963)$

The fuzzy success memberships and fuzzy failure memberships are depicted in Fig. 2 and Fig. 3 respectively. The fuzzy reliability is plotted in Fig. 4. From Fig. 4, we observe that the fuzzy reliability is reasonable for the fuzzy success and fuzzy failure.

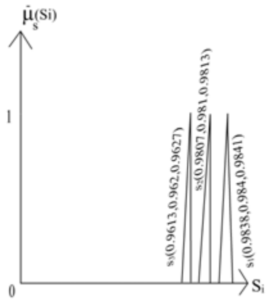

Figure 2. Fuzzy Success

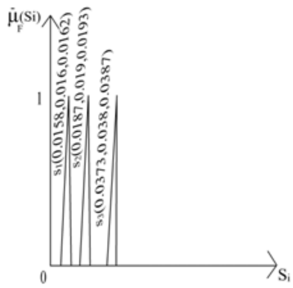

Figure 3. Fuzzy Failure

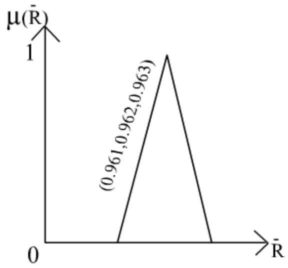

Figure 4. Fuzzy Reliability

Conclusions

We have presented a new method for finding fuzzy system reliability using fuzzy profust reliability theory and we have applied the above method for our website www.ssn.edu.in and obtained the results as a triangular fuzzy number. Since in most of the real life system, the system states are very often fuzzy, the above constructed method can be applied for all the models that are modeled as a fuzzy probabilistic semi-Markov model.

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